MATH 9C **Sample Midterm Exam**

Remember: You must show every step of your work to receive credit.

1. Using the appropriate theorems, find the limit if the sequence converges or show that it diverges.

(a) $\left(\dfrac{n}{n+2}\right)^n$ **(b)** $\dfrac{n^2+(-1)^n}{n^2}$

2. Find the sum of the series if it converges, or show that it diverges.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^n}{2^n}$ **(b)** $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2^n}+\frac{(-1)^n}{5^n}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{4}{(2n-1)(2n+1)}$ **(d)** $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{n}{n+1}\right)$

3. Express $0.\overline{414} = 0.414\ 414\ 414\ \ldots$ as a proper fraction.

4. Apply the indicated test to determine convergence or divergence of the given series. (Give the statement of each test before applying it.)

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\,(1+\ln^2 n)}$ (Integral Test)

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1+\sin n}{n^3}$ (Direct Comparison Test)

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(\ln n)^2}$ (Limit Comparison Test)

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{n\,\ln n}{2^n}$ (Ratio Test)

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)^n}{(2^n)^3}$ (Root Test)

5. Consider the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{10^n}.$$

(a) Give the first three partial sums of this series?
(b) Is this series convergent? Is it absolutely convergent?
(c) What is the error if this series is approximated by its third partial sum?