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Lie n-algebras, supersymmetry, & division algebras

Papers: Division algebras and supersymmetry I & II.

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Puzzle

- The only normed division algebras are $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ & $\mathbb{O}$. They have dimension $K = 1, 2, 4, 8$.

- The classical superstring makes sense in spacetimes of dimension $K+2 = 3, 4, 6, 10$.

- The classical super-2-brane makes sense in spacetimes of dimension $K+3 = 4, 5, 7, 11$.

Why? Spinor identities:

Setup:

- V a real vector space w/ nondegen. quadratic form.

  Think Minkowski spacetime.

- S a spinor rep of $\mathcal{A}O(V)$.
- $\mathrm{Sym}^2 S \xrightarrow{[\,,\,]} V$ an $\mathcal{A}O(V)$-equivariant map.

For superstrings: $K+2 = 3, 4, 6, 10$.

Need: $[\psi, \psi]\psi = 0 \quad \forall\, \psi \in S$.

For 2-branes: $K+3 = 4, 5, 7, 11$.

Need: $[\psi, [\psi, \psi]\psi] = 0 \quad \forall\, \psi \in S$.

Role of the division algebras

Let $\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$.

For superstrings: $k+2 = 3, 4, 6, 10$.

- $V = \mathfrak{h}_2(\mathbb{K}) = \left\{ \begin{bmatrix} t+x & \bar{y} \\ y & t-x \end{bmatrix} : t, x \in \mathbb{R}, y \in \mathbb{K} \right\}$
- $S = \mathbb{K}^2$.
- $[\,,\,] : \mathrm{Sym}^2 S \to V$ has a nice formula!
- $[\psi, \psi]\psi = 0$ is easy!

For 2-branes: $k+3 = 4, 5, 7, 11$.

- $V \subseteq \mathbb{K}[4]$, 4×4 matrices over $\mathbb{K}$.
- $S = \mathbb{K}^4$.
- $[\,,\,] : \mathrm{Sym}^2 S \to V$ has a nice formula!
- $[\psi, [\psi, \psi]\psi] = 0$ is easy!

Cocycle Conditions

We have the superPoincaré algebra:

$$\mathfrak{siso}(V) = \mathfrak{so}(V) \ltimes (V \oplus S)$$

For superstrings: $k+2 = 3, 4, 6, 10.$

- $\exists \, [\alpha] \in H^3_{CE}(\mathfrak{siso}(V))$
- $d\alpha = 0 \iff [\psi, \psi]\psi = 0$

For 2-branes: $k+3 = 4, 5, 7, 11.$

- $\exists \, [\beta] \in H^4_{CE}(\mathfrak{siso}(V))$
- $d\beta = 0 \iff [\psi, [\psi, \psi]\psi] = 0$

Lie n-algebras

Thm (Baez - Crans - Huerta)

Given a $(n+1)$-cocycle ω on $\mathfrak{g}$, there is a Lie n-superalgebra:

$$\mathfrak{g} \leftarrow 0 \leftarrow \cdots \leftarrow 0 \leftarrow \mathbb{R}$$

where

- $[\cdot,\cdot]$ is the bracket on $\mathfrak{g}$.
- $[\cdot, \ldots, \cdot] = \omega$

Thm (superstrings)

In dim $k+2 = 3, 4, 6, 10$, $\exists$ a Lie 2-superalgebra built from α.

Thm (2-branes)

In dim $k+3 = 4, 5, 7, 11$, $\exists$ a Lie 3-superalgebra built from β.