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In[1]:= << LinearAlgebra`MatrixManipulation`
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$$\text{In}[2]:= \mathbf{m} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\text{Out}[2]= \{\{1, 1\}, \{1, 0\}, \{1, 0\}\}$$

$$\text{In}[3]:= \{\text{nrows}, \text{ncols}\} = \text{Dimensions}[\mathbf{m}]$$

$$\text{Out}[3]= \{3, 2\}$$

$$\text{In}[4]:= \mathbf{t1} = \text{Flatten}[\text{Table}[\{\mathbf{r}, \mathbf{c}\}, \{\mathbf{r}, 1, \text{nrows}\}, \{\mathbf{c}, 1, \text{ncols}\}], 1];$$

$$\text{In}[5]:= \mathbf{t2} = \text{Select}[\mathbf{t1}, \mathbf{m}[[\#\{1\}], \#\{2\}]] == 1 \&;$$

$$\text{In}[6]:= \mathbf{t3} = \text{Map}[\text{Apply}[\mathbf{v}, \#] \&, \mathbf{t2}];$$

$$\text{In}[7]:= \text{req}[\mathbf{m}_1, \mathbf{r}_1] := (\text{Sum}[\mathbf{v}[\mathbf{r}, \mathbf{c}] * \mathbf{m1}[[\mathbf{r}, \mathbf{c}]], \{\mathbf{c}, 1, \text{ncols}\}] == k)$$

$$\text{In}[8]:= \text{ceq}[\mathbf{m}_1, \mathbf{c}_1] := (\text{Sum}[(1/\mathbf{v}[\mathbf{r}, \mathbf{c}]) * \mathbf{m1}[[\mathbf{r}, \mathbf{c}]], \{\mathbf{r}, 1, \text{nrows}\}] == k)$$

$$\text{In}[9]:= \text{Solve}[\text{Join}[\text{Table}[\text{req}[\mathbf{m}, \mathbf{r}], \{\mathbf{r}, 1, \text{nrows}\}], \text{Table}[\text{ceq}[\mathbf{m}, \mathbf{c}], \{\mathbf{c}, 1, \text{ncols}\}]], \text{Append}[\mathbf{t3}, k]]$$

$$\text{Out}[9]= \left\{ \begin{aligned} \mathbf{v}[1, 2] &\rightarrow \frac{1}{2} \left(-2 \sqrt{2 - \sqrt{2}} - \sqrt{2 (2 - \sqrt{2})} \right), k \rightarrow -\sqrt{2 - \sqrt{2}}, \\ \mathbf{v}[1, 1] &\rightarrow \sqrt{\frac{1}{2} (2 - \sqrt{2})}, \mathbf{v}[2, 1] \rightarrow -\sqrt{2 - \sqrt{2}}, \mathbf{v}[3, 1] \rightarrow -\sqrt{2 - \sqrt{2}}, \\ \mathbf{v}[1, 2] &\rightarrow \frac{1}{2} \left(2 \sqrt{2 - \sqrt{2}} + \sqrt{2 (2 - \sqrt{2})} \right), k \rightarrow \sqrt{2 - \sqrt{2}}, \mathbf{v}[1, 1] \rightarrow -\sqrt{\frac{1}{2} (2 - \sqrt{2})}, \\ \mathbf{v}[2, 1] &\rightarrow \sqrt{2 - \sqrt{2}}, \mathbf{v}[3, 1] \rightarrow \sqrt{2 - \sqrt{2}}, \{ \mathbf{v}[1, 2] \rightarrow \frac{1}{2} \left(2 \sqrt{2 + \sqrt{2}} - \sqrt{2 (2 + \sqrt{2})} \right), \\ k &\rightarrow \sqrt{2 + \sqrt{2}}, \mathbf{v}[1, 1] \rightarrow \sqrt{\frac{1}{2} (2 + \sqrt{2})}, \mathbf{v}[2, 1] \rightarrow \sqrt{2 + \sqrt{2}}, \mathbf{v}[3, 1] \rightarrow \sqrt{2 + \sqrt{2}}, \\ \mathbf{v}[1, 2] &\rightarrow \frac{1}{2} \left(-2 \sqrt{2 + \sqrt{2}} + \sqrt{2 (2 + \sqrt{2})} \right), k \rightarrow -\sqrt{2 + \sqrt{2}}, \\ \mathbf{v}[1, 1] &\rightarrow -\sqrt{\frac{1}{2} (2 + \sqrt{2})}, \mathbf{v}[2, 1] \rightarrow -\sqrt{2 + \sqrt{2}}, \mathbf{v}[3, 1] \rightarrow -\sqrt{2 + \sqrt{2}} \} \end{aligned} \right\}$$

$$\text{In}[10]:= \text{Eigenvalues}[\text{BlockMatrix}[\begin{array}{c|c} \text{Table}[0, \{\text{ncols}\}, \{\text{ncols}\}] & \text{Transpose}[\mathbf{m}] \\ \hline \mathbf{m} & \text{Table}[0, \{\text{nrows}\}, \{\text{nrows}\}] \end{array}]]]$$

$$\text{Out}[10]= \{-\sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2}}, -\sqrt{2 - \sqrt{2}}, \sqrt{2 - \sqrt{2}}, 0\}$$

$$\text{In}[11]:= \text{SingularValueList}[\mathbf{N}[\mathbf{m}]]$$

$$\text{Out}[11]= \{1.84776, 0.765367\}$$