University of Michigan
Student Name:

Math 105 Quiz 1

Section 101 - Spring 2011
Instructor: Jose Gonzalez
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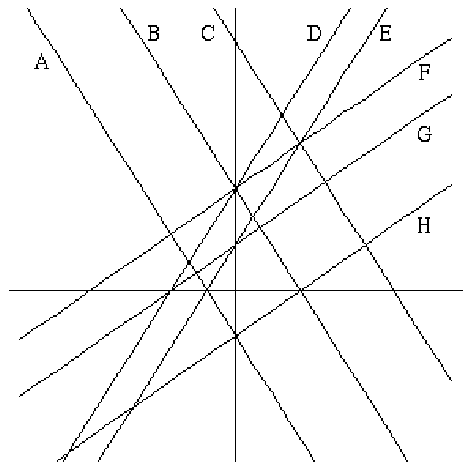
(1) (8 pts.) Choose the equation defining each line in this rectangle, by matching its letter to one of the formulas that follow. The figure *is* drawn to scale.



___ $y = 220 - \frac{3}{2}x$ ___ $y = 90 + \frac{3}{2}x$ ___ $y = 90 - \frac{3}{2}x$ ___ $y = 90 + \frac{2}{3}x$

___ $y = 40 + \frac{3}{2}x$ ___ $y = 40 + \frac{2}{3}x$ ___ $y = -40 - \frac{3}{2}x$ ___ $y = -40 + \frac{2}{3}x$

Solution We first notice that C has a vertical intercept of 220; that B, D, F have vertical intercepts of 90; that E, G have vertical intercepts of 40; and that A, H have vertical intercepts of -40. Next, we notice that the slope of D is larger than the slope of F and both are positive, and that the slope of B is negative; that the slope of E is larger than the slope of G; and that the slope of A is larger than the slope of H. Therefore the graphs corresponding to the formulas are:

C $y = 220 - \frac{3}{2}x$ D $y = 90 + \frac{3}{2}x$ B $y = 90 - \frac{3}{2}x$ F $y = 90 + \frac{2}{3}x$

E $y = 40 + \frac{3}{2}x$ G $y = 40 + \frac{2}{3}x$ A $y = -40 - \frac{3}{2}x$ H $y = -40 + \frac{2}{3}x$

(2) (12 pts.) Sunnyside Cars, a local car dealership, has found that there is a linear relationship between the amount of money it spends on advertising and the number of cars it sells in a month. Let N be the monthly car sales as a function of x, the thousands of dollars spent on advertising.

(a) (3 pts.) Give a practical interpretation of the vertical intercept of N.

Answer: The vertical intercept $N(0)$ represents the amount of cars that the company would sell if it does not spend money on advertising.

(b) (3 pts.) Give a practical interpretation of $N^{-1}(400)$.

Answer: The quantity $N^{-1}(400)$ represents the number of thousands of dollars that the company needs to spend on advertising in order to sell 400 cars.

When Sunnyside spent 50 thousand dollars on advertising, it sold 540 cars a month. Moreover, for each additional 5 thousand dollars spent, the company sells 40 more cars a month.

(c) (3 pts.) Determine a formula for $N(x)$.

Answer: We have that the points $(50, 540)$ and $(55, 580)$ are on the graph of this linear function. Then the slope of this line is $\frac{580-540}{55-50} = 8$. Therefore using the point-slope form for the equation of a linear function we get $N(x) = 8(x - 50) + 540$.

(d) (3 pts.) Determine the value of $N^{-1}(400)$.

Answer: We need to solve for x in the equation $N(x) = 400$, or equivalently on $8(x-50)+540 = 400$. One gets $8x = 400 + 400 - 540 = 260$, and then $x = 32.5$. Therefore, the value of $N^{-1}(400)$ is 32500 dollars.

(3) (10 pts.) Let

$$y = f(x) = \frac{1}{\sqrt{6 + x - x^2}},$$

where x and y take only real values. Please **answer** each of the following questions, and in each case include one or two sentences **explaining** how you obtained your answer.

(a) (2 pts.) Factor the quadratic function $g(x) = 6 + x - x^2$ and find its zeros.

Answer: $g(x) = 6 + x - x^2 = -[x^2 - x - 6] = -(x-3)(x+2)$ (you can also include the minus sign in any of the factors and change the signs accordingly). The zeroes of g are $x = 3$ and $x = -2$.

(b) (2 pts.) What is the domain of f? (Hint: Look at the graphs of both f and g)

Answer: The domain of f is the set of x values that make the quantity $6 + x - x^2$ positive. By looking at the graph of g and using part (b), we see that the answer is the interval between the zeroes of g, so the domain of f is the interval $(-2, 3)$.

(c) (2 pts.) What is the range of f?

Answer: By looking at the graph of the function f, we see that its range is the interval that goes from the smallest value of f (and that value is included) to infinity. The smallest value of f happens when the quantity $6 + x - x^2$ is the largest possible, and by looking at the graph of $g(x) = 6 + x - x^2$ we see that this happens in the midpoint of the zeroes of g. We find that this midpoint is $x = 0.5$, and that $f(0.5) = \frac{1}{2.5} = 0.4$. Then the range of f is the interval $[0.4, \infty)$. Note: A close estimation of the correct value 0.4 using the graph will be accepted.

(d) (2 pts.) Describe the concavity of f.

From the graph of f as seen on the calculator we deduce that it is concave up.

(e) (2 pts.) Is f invertible?

From the graph of f as seen on the calculator, we notice that there are pairs of distinct inputs that produce the same output (for instance $x = 0$ and $x = 1$ produce the same output). Then the function f is not invertible.