University of Michigan
Student Name:

Math 105 Quiz 2

Section 101 - Spring 2011
Instructor: Jose Gonzalez
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(1) (10 pts.) As you plan your vacation around the world to celebrate your successful completion of Jose's class, you realize that you are 1161 dollars short. *Hands Up Bank* offers to lend you the money with a nominal annual interest rate of 20%.

Taken from www.slate.com

At that moment *M. C. Hammerhead* the local loan shark, approaches you and tells you that he will give you a much better deal. He will kindly lend you the same money with a nominal annual interest rate of only 19% (how kind!). *Hands Up Bank* compounds their interest annually, but as everybody knows M. C. compounds his interest continuously.

(a) (3 pts.) Give a formula for $B(t)$, the amount of money you would owe to *Hands Up Bank* after t years if you take their loan.

Answer: $B(t) = 1161 \cdot 1.2^t$ dollars.

(b) (4 pts.) Give a formula for $S(t)$, the amount of money you would owe to M. C. after t years if you decide to do business with such fine gentleman.

Answer: $S(t) = 1161 \cdot e^{0.19t}$ dollars.

(c) (3 pts.) If you plan to pay your loan in a single payment five years after receiving the money, which loan gives you the better deal?

Answer: After five years, if you took the bank loan you would owe the bank $B(5) = 1161 \cdot 1.2^5 = 2888.94$ dollars, and if you took M. C.'s loan you would owe him $S(t) = 1161 \cdot e^{0.19 \cdot 5} = 3002$ dollars. Then, to save money you should take the bank loan.

(2) (10 pts.) Suppose 2 mg of a drug is injected into a person's bloodstream. As the drug is metabolized, the quantity diminishes at the continuous rate of 4% per hour.

(a) (3 pts.) Find a formula for $Q(t)$, the quantity of the drug remaining in the body after t hours.

Answer: $Q(t) = 2 \cdot e^{-0.04t}$ mg.

(b) (3 pts.) By what percent does the drug level decrease during any given hour?

Answer: We have that $e^{-0.04} = 0.96079$. Then $Q(t) = 2 \cdot 0.96079^t$ mg. Hence after each hour the level of the drug gets multiplied by 0.96079. Then on any given hour the level changes by a factor of $0.96079 - 1 = -0.03921$, this is, on any given hour the drug level decreases by 3.921%.

(c) (4 pts.) The person must receive an additional 2 mg of the drug once its level has diminished to 0.25 mg. When must the person receive the second injection.

Answer: We need to find the time t so that $Q(t) = 0.25$. So we need to solve $2 \cdot e^{-0.04t} = 0.25$. We get $e^{-0.04t} = 0.125$, and then we take natural logarithm to get $-0.04t = \ln 0.125$. Then the person needs to receive a second injection after $t = \frac{\ln 0.125}{-0.04} = 51.986$ hours.

(3) (10 pts.) Please answer each of the following questions, in each case you must **show your work**.

(a) (3 pts.) Find the equation of the exponential function whose graph passes through the points $(-30, 200)$ and $(20, 60)$.

Answer: Let $f(t) = a \cdot b^x$ be the exponential we are looking for. Then $200 = a \cdot b^{-30}$ and $60 = a \cdot b^{20}$. From the first equation $a = \frac{200}{b^{-30}}$, and we substitute in the second equation to obtain $60 = \frac{200}{b^{-30}} \cdot b^{20}$. Then $\frac{60}{200} = b^{50}$, and therefore $b = \left(\frac{60}{200}\right)^{1/50} = 0.9762$. Then $a = \frac{200}{0.9762^{-30}} = 97.1187$. Then the equation we are looking for is $f(t) = 97.1187 \cdot 0.9762^x$.

(b) (3 pts.) Express the exponential function $g(x) = 7 \cdot 1.23^x$ in the form $g(x) = a \cdot e^{kx}$. What are its growth factor and its continuous growth rate?

Answer: We need to find k so that $e^k = 1.23$. Taking natural logarithm on both sides of this equation, we get $k = \ln 1.23 = 0.207014$. Therefore $g(x) = 7 \cdot e^{0.207014 \cdot x}$. The growth factor of this exponential function is 1.23 and its continuous growth rate is 0.207014.

(c) (4 pts.) Solve algebraically the equation $5 \cdot 3^{2x} = 2 \cdot 7^x$.

Answer: The equation is equivalent to $\frac{3^{2x}}{7^x} = \frac{2}{5}$. And this is equivalent to $\left(\frac{9}{7}\right)^x = \frac{2}{5}$. Taking natural logarithm to both sides of this equality we get that $x \cdot \ln \frac{9}{7} = \ln \frac{2}{5}$. Then

$$x = \frac{\ln \frac{2}{5}}{\ln \frac{9}{7}},$$

and then $x = -3.646$.