University of Michigan
Student Name:

Math 105 Quiz 3

Section 101 - Spring 2011
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(1) (10 pts.) An espresso stand finds that its weekly profit is a function of the price, x, it charges per cup. If x is in dollars, the weekly profit is $P(x) = -3400x^2 + 10200x - 6800$ dollars. Solve the following questions algebraically showing all your work. Note: Answers obtained from a graph would get no credit.

(a) (3 pts.) Express the quadratic function that gives the profit in its vertex form.

Solution:

$$\begin{aligned}
P(x) &= -3400x^2 + 10200x - 6800 \\
&= -3400[x^2 - 3x + 2] \\
&= -3400[x^2 - 3x + (\frac{3}{2})^2 - (\frac{3}{2})^2 + 2] \\
&= -3400[(x - \frac{3}{2})^2 - (\frac{9}{4}) + 2] \\
&= -3400[(x - \frac{3}{2})^2 - (\frac{1}{4})] \\
&= -3400(x - \frac{3}{2})^2 + 3400(\frac{1}{4}) \\
&= -3400(x - \frac{3}{2})^2 + 850.
\end{aligned}$$

(b) (2 pts.) What is the maximum possible weekly profit?

Solution: The maximum weekly profit is equal to the y-coordinate of the vertex of this opening downward parabola. So, the maximum possible weekly profit is 850 dollars.

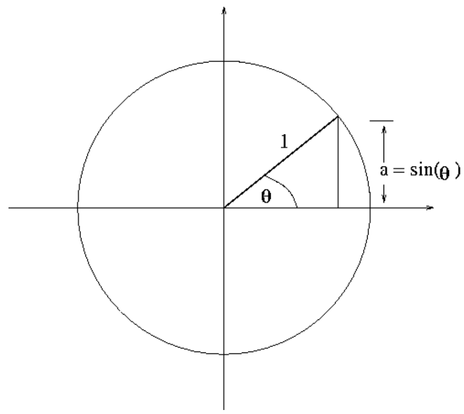
(c) (2 pts.) What price per cup that produces that maximum profit?

Solution: The maximum weekly profit is occurs when the price per cup of coffee is equal to the x-coordinate of the vertex of this opening downward parabola. So, the price per cup of coffee that produces the maximum possible weekly profit is $\frac{3}{2} = 1.5$ dollars per cup of coffee.

(d) (3 pts.) Find the possible prices per cup for which the espresso stand would break even.

Solution: To find the prices for which the espresso stand would break even, we need to solve the equation $-3400x^2 + 10200x - 6800 = 0$. Using either factorization to get $-3400(x-1)(x-2) = 0$, or the formula for the zeroes of a quadratic to get $x = \frac{-10200 \pm \sqrt{10200^2 - 4 \cdot (-3400) \cdot (-6800)}}{2(-3400)}$, we see that the zeroes of this quadratic are $x = 1$ and $x = 2$. Therefore the espresso stand breaks even when the price is either 1 dollar per cup of coffee or 2 dollars per cup of coffee.

(2) (10 pts.) Let θ be an angle in the first quadrant, and suppose $\sin(\theta) = a$. Evaluate the following expressions in terms of a.



(a) (2 pts.) $\sin(-\theta) =$

Answer: Using the graph we see that $\sin(-\theta) = -\sin(\theta) = -a$.

(b) (2 pts.) $\cos(\theta) =$

Answer: The right triangle in the picture has angles θ, $\pi/2 - \theta$ and $\pi/2$. By the Pythagorean Theorem, we have that the adjacent side to the angle θ (which is the opposite side to the angle $\pi/2 - \theta$) is equal to $\sqrt{1-a^2}$. Then $\cos(\theta) = \frac{\sqrt{1-a^2}}{1} = \sqrt{1-a^2}$.

(c) (2 pts.) $\tan(\theta + \pi) =$

Answer: After drawing the angle $\theta+\pi$, we notice that $\sin(\theta+\pi) = -\sin(\theta)$ and $\cos(\theta+\pi) = -\cos(\theta)$. Therefore $\tan(\theta + \pi) = \frac{\sin(\theta+\pi)=}{\cos(\theta+\pi)} = \frac{-\sin(\theta)=}{-\cos(\theta)} = \frac{a}{\sqrt{1-a^2}}$.

(d) (2 pts.) $\sin(3\pi - \theta) =$

Answer: Using the graph we see that $\sin(3\pi - \theta) = \sin(\pi - \theta) = \sin(\theta) = a$.

(e) (2 pts.) $\sin(\theta + \frac{\pi}{2}) =$

Answer: Using the graph we see that $\sin(\theta + \frac{\pi}{2}) = \sin(\frac{\pi}{2} - \theta) = \cos(\theta) = \sqrt{1-a^2}$.

(3) (10 pts.) Find formulas for the functions described in the following statements.

(a) (5 pts.) A ferris wheel is 20 meters in diameter and boarded at ground level. The wheel makes one full rotation every 6 minutes, and at time $t = 0$ you are at the 3 o'clock position and ascending. Let $f(t)$ denote your height (in meters) above ground at t minutes. Find a formula for $f(t)$.

Solution Since the diameter is 20 meters, your height would be 10 meters when you are halfway up the ferris wheel, so the midline is $y = k = 10$. Your maximum height is 20 meters, so the amplitude is $A = 10$. The period is 6 minutes, so the constant $B = 2\pi/6$. At $t = 0$, the 3 o'clock position would be halfway up the ferris wheel. We want the graph to be at its midline when $t = 0$, so we want to use the sine function. We also want the graph to begin increasing at a first since the ferris wheel is ascending, so we need to use a function of the form $f(t) = A\sin(Bt) + k$:

$$f(t) = 10\sin\left(\frac{2\pi}{6}t\right) + 10.$$

(b) (5 pts.) The height in centimeters of the tip of the minute hand on a vertical clock is a function, $g(t)$, of the time, t, in minutes after midnight. The minute hand is 30 cm long, and the middle of the clock face is 244 cm above the ground. Find a formula for $g(t)$.

Solution Since the middle of the clock face is 244 cm above the ground then the midline is $y = k = 244$. Since the length of the minute hand is 30 cm then the amplitude is $A = 30$. Since the minute hand completes a loop every 60 minutes, then the period is 60 and the constant $B = 2\pi/60$. The time starts when the minute hand is at its maximum height, so we want to use a cosine function with equation of the form $g(t) = A\cos(Bt) + k$:

$$g(t) = 30\cos\left(\frac{2\pi}{60}t\right) + 244.$$