University of British Columbia
Student Name:

Math 184 Quiz 1

Team 922 - Summer 2013
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(1) Suppose that the functions $f(x)$ and $g(x)$ are differentiable. Suppose also that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$ and $g'(1) = 4$. Compute the following, and simplify your answers showing all your work.

a) Find $h'(1)$, if $h(x) = 2f(x) - 3g(x)$.

Solution

$$\begin{aligned} h'(x) &= 2f'(x) - 3g'(x) \\ h'(1) &= 2f'(1) - 3g'(1) \\ h'(1) &= 2 \cdot 3 - 3 \cdot 4 = -6 \end{aligned}$$

b) Find $n'(1)$, if $n(x) = x^2 f(x) + \frac{g(x)}{x}$.

Solution

$$\begin{aligned} n'(x) &= 2xf(x) + x^2 f'(x) + \frac{g'(x) \cdot x - g(x)}{x^2} \\ n'(1) &= 2 \cdot 1 \cdot f(1) + 1^2 f'(1) + \frac{g'(1) \cdot 1 - g(1)}{1^2} \\ n'(1) &= 2 \cdot 1 \cdot 1 + 1^2 \cdot 3 + \frac{4 \cdot 1 - 2}{1^2} = 7 \end{aligned}$$

c) Find the equation of the tangent line to the graph of the function $h(x) = 2f(x) - 3g(x)$ in part (a), at the point corresponding to $x = 1$.

Solution

When $x = 1$: $h(1) = 2f(1) - 3g(1) = 2 \cdot 1 - 3 \cdot 2 = -4$. Then, we need the equation of the line passing through $(1, h(1)) = (1, -4)$ with slope $h'(1) = -6$:

$$\begin{aligned} y + 4 &= -6(x - 1) \\ y &= -6x + 2 \end{aligned}$$

(2) Compute the following limits showing all your work:

a) $\lim\limits_{x \to -2} \dfrac{x^2 + 4x + 4}{x^2 - 4}$

Solution

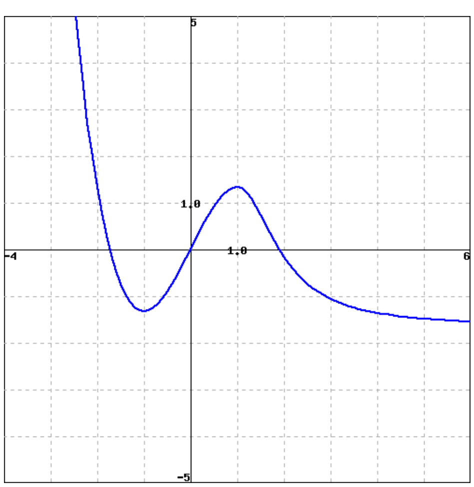
$$\lim_{x \to -2} \frac{x^2 + 4x + 4}{x^2 - 4} = \lim_{x \to -2} \frac{(x+2)^2}{(x+2)(x-2)} = \lim_{x \to -2} \frac{x+2}{x-2} = \frac{-2+2}{-2-2} = 0.$$

b) $\lim\limits_{x \to 1} \dfrac{x - 1}{\sqrt{x} - 1}$

Solution

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x} - 1} = \lim_{x \to 1} (\sqrt{x} + 1) = \sqrt{1} + 1 = 2.$$

(3) For the function $g(x)$ whose graph is given below, arrange the following values in increasing order: **a)** $g'(-2)$ **b)** $g'(0)$ **c)** $g'(2)$ **d)** $g'(4)$. No explanation required.



Solution: **a**) $g'(-2)$ **c**) $g'(2)$ **d**) $g'(4)$ **b**) $g'(0)$.

To get this answer we interpreted each of these the derivatives as the slope of the tangent line to the graph at the corresponding point, and then estimated their values from the graph.

(4) Complete the statements:

a) The intermediate value theorem says that if $f(x)$ is a ___**Continuous**___ function in the interval $[a, b]$, and the number L is between ___$f(a)$___ and ___$f(b)$___, then there exists c between ___a___ and ___b___ such that ___$f(c) = L$___.

b) If $f(x)$ is a function defined on an interval around the point $x = a$, we defined that the function is continuous at the point $x = a$, when the following three conditions hold: ___$f(a)$___ exists, ___$\lim_{x \to a} f(x)$___ exists, and in addition ___$\lim_{x \to a} f(x) = f(a)$___.

(5) Sketch the graph of derivative $g'(x)$ of the function $g(x)$ given in the graph below. (Important features are: where the derivative does not exist, and where it is either positive, negative or zero.)