University of British Columbia
Student Name:

Math 184 Quiz 2

Team 922 - Summer 2013
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(1) Chain Rule.

a) Find the derivative of $f(x) = [\ln(1+e^x)]^5$.

Solution:

$$f'(x) = 5 \cdot [\ln(1+e^x)]^4 \cdot \frac{1}{1+e^x} \cdot e^x.$$

b) Find the derivative of $g(x) = \ln(\sin(3x^4 - 7x))$.

Solution:

$$g'(x) = \frac{1}{\sin(3x^4 - 7x)} \cdot \cos(3x^4 - 7x) \cdot (12x^3 - 7).$$

c) If $h(x) = \sqrt{7 + 6f(x)}$, where $f(1) = 3$ and $f'(1) = 5$, find the tangent line to the graph of $h(x)$ at the point where $x = 1$.

Solution: We need to find the tangent line through the point $(1, h(1)) = (1, \sqrt{7+6f(x)}) = (1, \sqrt{25}) = (1, 5)$. This tangent line has slope $h'(1)$. We have that $h'(x) = \frac{1}{2}(7+6f(x))^{-1/2}(6f'(x))$. Then,

$$h'(1) = \frac{1}{2}(7+6f(1))^{-1/2}(6f'(1)) = \frac{1}{2}(7+6\cdot 3)^{-1/2}(6f'(1)) = \frac{6\cdot 5}{2\cdot\sqrt{25}} = 3.$$

Therefore, the equation of the line is $y - 5 = 3(x-1)$, that is $y = 3x + 2$.

(2) Implicit Differentiation.

a) If $5\sin(x-y) = 2y\sin(x)$, compute $\frac{dy}{dx}$ using implicit differentiation.

Solution:

$$\begin{aligned}
5\cos(x-y)\cdot(1-\frac{dy}{dx}) &= 2\frac{dy}{dx}\sin(x) + 2y\cdot\cos(x)\\
5\cos(x-y) - 5\cos(x-y)\cdot\frac{dy}{dx} &= 2\frac{dy}{dx}\sin(x) + 2y\cdot\cos(x)\\
-5\cos(x-y)\cdot\frac{dy}{dx} - 2\frac{dy}{dx}\sin(x) &= 2y\cdot\cos(x) - 5\cos(x-y)\\
\frac{dy}{dx}\cdot[-5\cos(x-y) - 2\sin(x)] &= 2y\cdot\cos(x) - 5\cos(x-y)\\
\frac{dy}{dx} &= \frac{2y\cdot\cos(x) - 5\cos(x-y)}{-5\cos(x-y) - 2\sin(x)}.
\end{aligned}$$

b) Use implicit differentiation to find the equation of the tangent line to the curve $xy^3 + xy = 14$ at the point (7,1).

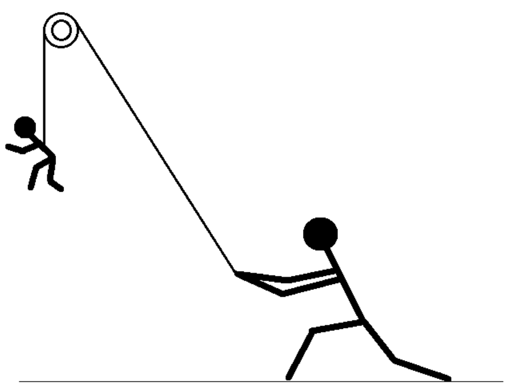
Solution: Using implicit differentiation we have

$$\begin{aligned}
y^3 + x\cdot 3y^2\frac{dy}{dx} + y + x\cdot\frac{dy}{dx} &= 0\\
x\cdot 3y^2\frac{dy}{dx} + x\cdot\frac{dy}{dx} &= -y^3 - y\\
\frac{dy}{dx}\cdot[3xy^2 + x] &= -y^3 - y\\
\frac{dy}{dx} &= \frac{-y^3 - y}{3xy^2 + x}.
\end{aligned}$$

The slope of the tangent line is $\frac{dy}{dx}|_{(7,1)} = \frac{-1-1}{21+7} = -\frac{1}{14}$. Therefore, the equation of the desired tangent line is $y - 1 = -\frac{1}{14}(x-7)$, which is the same as $y = -\frac{1}{14}x + \frac{3}{2}$.

(3) Related Rates.

A child climbs the rock-climbing wall in a gym. For safety, his father stays on the ground and holds the kid with a rope. The father holds one end of the rope, which then goes to a pulley directly above the kid, and the other end of the rope is attached to the kid's body. The father decides to pull the kid to the top by holding the rope and walking away from the wall with it. If the rope is $32ft$ long, the pulley is $20ft$ from the floor and the father holds the end of the rope $4ft$ above the ground while walking away at a speed of $3\frac{ft}{s}$, at what speed does the boy rise when his father is $12ft$ away from the wall? [The picture is not drawn to scale. In this problem you should treat the kid and the pulley as dots on the wall, and the father as a dot where the rope ends $4ft$ above the ground.] (In case it is useful: $12^2 + 16^2 = 144 + 256 = 400 = 20^2$.)



Let x be the distance in feet of the father to the wall and let y the distance in feet of the kid to the pulley. Let t be the time measured in seconds. We have that $\frac{dx}{dt} = 3\frac{ft}{s}$. To answer the question we need to find $\frac{dy}{dt}$ when $x = 12ft$. The triangle with vertices on the end of the rope in the fathers hand, the point on the wall on the same horizontal line and the pulley is a right triangle with legs x and 16 and with hypotenuse $32 - y$. Then we have $(32 - y)^2 = x^2 + 16^2$.

Implicit differentiation with respect to time gives:

$$2(32 - y)(-\frac{dy}{dt}) = 2x\frac{dx}{dt}$$
$$\frac{dy}{dt} = -\frac{x}{32 - y}\frac{dx}{dt}.$$

When $x = 12\,ft$, we have that $(32 - y)^2 = 12^2 + 16^2 = 400 = 20^2$. Then $32 - y = \pm 20$. Then $y = 12$ or $y = 52$, but the second option is discarded because y is at most equal to the length of the rope which is $32ft$. Then, when $x = 12$ we have $y = 12$. We substitute everything in the expression for $\frac{dy}{dt}$ to obtain:

$$\frac{dy}{dt} = -\frac{12}{32 - 12}3 = -\frac{36}{20} = -1.8\,\frac{ft}{s}.$$

Therefore, the boy is rising at a speed of $1.8\,\frac{ft}{s}$.