University of British Columbia
Student Name:

Math 184 Quiz 3

Team 922 - Summer 2013
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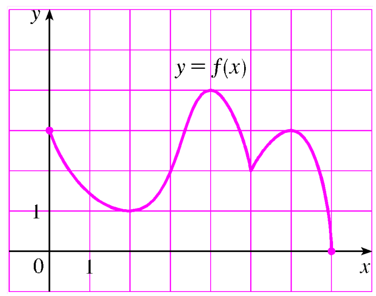
(1) (8pts) The function $f(x)$ is given by the following graph.



a) (2pts) What are the critical points of $f(x)$ on the interval $(0, 7)$?

Solution $x = 2, 4, 5, 6$.

b) (2pts) What are the local maxima of $f(x)$ on the interval $(0, 7)$?

Solution $x = 4, 6$.

c) (2pts) What are the local minima of $f(x)$ on the interval $(0, 7)$?

Solution $x = 2, 5$.

d) (2pts) What are the absolute maxima and absolute minima of $f(x)$ on the interval $[0, 7]$?

Solution Absolute maximum $f(4) = 4$ and absolute minimum at $f(7) = 0$.

(2) (6pts) For the function $f(x) = x^4 - 4x^3 + 4x^2 - 4 = (x^2 - 2x + 2)(x^2 - 2x - 2)$ one can easily see that $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$ and $f''(x) = 4(3x^2 - 3x + 2)$. Find the absolute maximum and the absolute minimum of $f(x)$ on the interval $[-2, 3]$.

Solution The critical points are $x = 0, 1, 2$. Since $f(x)$ is a continuous function on the finite closed interval $[-2, 3]$, the candidates to absolute maximum and the absolute minimum are the critical points and the end points. We have that $f(0) = -4$, $f(1) = -3$, $f(2) = -4$, $f(-2) = 60$ and $f(3) = 5$. Therefore, the required absolute maximum is $f(-2) = 60$ and the required absolute minimum is $f(0) = f(2) = -4$.

(3) (12pts) For the function $g(x) = xe^{-6x^2}$ one can easily see that $g'(x) = (1 - 12x^2)e^{-6x^2}$ and $g''(x) = 36x(2x - 1)(2x + 1)e^{-6x^2}$.

a) (3pts) Find the critical points of $g(x)$.

Solution The function $g'(x)$ is defined everywhere. If $g'(x) = 0$, then $(1 - 12x^2)e^{-6x^2} = 0$, so $1 - 12x^2 = 0$. Solving for x we find: $x = 1/\sqrt{12}$ or $x = -1/\sqrt{12}$, and these are the only two critical points of $g(x)$.

b) (3pts) Find the intervals where $g(x)$ is increasing and the intervals where $g(x)$ is decreasing.

Solution The function $g'(x)$ is continuous, so it cannot change sign except possibly at $x = 1/\sqrt{12}$ and $x = -1/\sqrt{12}$. We see that $g'(x)$ is positive on $(-1/\sqrt{12}, 1/\sqrt{12})$ and negative on $(-\infty, -1/\sqrt{12})$ and $(1/\sqrt{12}, \infty)$. Therefore, $g(x)$ is increasing on $(-1/\sqrt{12}, 1/\sqrt{12})$ and decreasing on $(-\infty, -1/\sqrt{12})$ and $(1/\sqrt{12}, \infty)$.

c) (3pts) Find the intervals where $g(x)$ is concave up and the intervals where $g(x)$ is concave down.

Solution The function $g''(x)$ is defined everywhere. If $g''(x) = 0$, then $6x(2x-1)(2x+1)e^{-6x^2} = 0$, so $6x(2x - 1)(2x + 1) = 0$. Solving for x we find: $x = -1/2$, $x = 0$ or $x = 1/2$.

The function $g''(x)$ is continuous, so it cannot change sign except possibly at $x = -1/2$, $x = 0$ and $x = 1/2$. We see that $g'(x)$ is positive on $(-1/2, 0)$ and $(1/2, \infty)$ and negative on $(-\infty, -1/2)$ and $(0, 1/2)$. Therefore, $g(x)$ is concave up on $(-1/2, 0)$ and $(1/2, \infty)$ and concave down on $(-\infty, -1/2)$ and $(0, 1/2)$.

d) (3pts) Find the inflection points of $g(x)$.

Solution From part c) the inflection points of $g(x)$ are $x = -1/2$, $x = 0$ or $x = 1/2$.

(4) (14pts) Solve the following exercises for the function $f(x)$ and its derivatives given below.

$$f(x) = \frac{x^3 + x^2 - 2x - 3}{x^2 - 3} \qquad f'(x) = \frac{(x^2-1)(x^2-6)}{(x^2-3)^2} \qquad f''(x) = \frac{2x(x^2+9)}{(x^2-3)^3}$$

a) (3pts) Find all x where $f'(x)$ is zero or does not exist.

Solution $\underline{f'(x) = 0}$: Then $(x^2-1)(x^2-6) = 0$; hence $(x+1)(x-1)(x+\sqrt{6})(x-\sqrt{6}) = 0$; and therefore $x = 1$, $x = -1$, $x = \sqrt{6}$, or $x = -\sqrt{6}$.

$\underline{f'(x)\ D.N.E.}$: Then $x^2 - 3 = 0$; hence $(x+\sqrt{3})(x-\sqrt{3}) = 0$; and therefore $x = \sqrt{3}$, or $x = -\sqrt{3}$.

b) (3pts) Find all x where $f''(x)$ is zero or does not exist.

Solution $\underline{f''(x) = 0}$: Then $2x(x^2+9) = 0$; but $x^2 + 9 > 0$ for all x; and therefore $x = 0$.

$\underline{f'(x)\ D.N.E.}$: Then $x^2 - 3 = 0$; hence $(x+\sqrt{3})(x-\sqrt{3}) = 0$; and therefore $x = \sqrt{3}$, or $x = -\sqrt{3}$.

c) (8pts) Find the coordinates of all local maxima, local minima and inflection points of the function $f(x)$. Show all your work. [Hint: Classify the points obtained in a) and b). But, discard those points where $f(x)$ is not defined.]

Solution We use the points in a) to find the intervals where $f'(x)$ is positive or negative, as $f'(x)$ is defined and continuous in the intervals between those points.

The function $f'(x)$ is positive on the intervals $(\infty, -\sqrt{6})$, $(-1, 1)$ and $(\sqrt{6}, \infty)$, and negative on the intervals $(-\sqrt{6}, -1)$ and $(1, \sqrt{6})$. Therefore $x = -\sqrt{6}$ and $x = 1$ are local maxima, and $x = -1$ and $x = \sqrt{6}$ are local minima.

Now, we use the points in b) to find the intervals where $g''(x)$ is positive or negative, as $f'(x)$ is defined and continuous in the intervals between those points.

The function $f''(x)$ is positive on the intervals $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$, and negative on the intervals $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$. But $f(x)$ is not defined when $x = \sqrt{3}$ or $x = -\sqrt{3}$. Therefore the only inflection point of $f(x)$ is $x = 0$.