(1) Solve for $a$:

$$m \cdot 3^{ab} = k \cdot e^a.$$  

(Here, $b$, $k$ and $m$ are constants.)

Solution

To solve for $a$, we take a logarithm of both sides. We'll use the natural logarithm because there’s a factor of $e^a$ on the right hand side of the equation. We get

$$\ln(m \cdot 3^{ab}) = \ln(k \cdot e^a),$$ or

$$\ln(m) + ab \ln(3) = \ln(k) + a.$$  

To solve for $a$, we subtract $a + \ln(m)$ from both sides, to get $ab \ln(3) - a = \ln(k) - \ln(m)$. Factoring out the $a$ on the left hand side, $a(b \ln(3) - 1) = \ln(k) - \ln(m)$, so

$$a = \frac{\ln(k) - \ln(m)}{b \ln(3) - 1}.$$  

(2) Suppose that the temperature of an office is given by $Q(t) = Q_0 + a \sin\left(\frac{\pi}{6}t\right)$, where $Q$ is in °F and $t$ in hours after 8AM. What are the meaning of $Q_0$ and $a$? What is the period of this function?

Solution

The parameter $Q_0$ is the midline, or average, temperature in the office. The maximum temperature deviation from this is $a$°F. The period of this function is $\frac{2\pi}{\frac{\pi}{6}} = 12$ hours.