University of Michigan
Student Name:

Math 115 Quiz 6

Section 9 - Winter 2011
Instructor: Jose Gonzalez
Date: March 11 / 2011

This quiz has two parts. please look on the back of the page.

(1) Suppose that your homework group has been asked to find all local minima of a function f on the interval $0 < x < 5$ and that both f' and f'' exist on this interval. The following discussion takes place among your group members. (You are the scribe.)

Manager: I took the derivative of f, and there's only one place on this interval where the derivative is equal to 0, namely when $x = 3$. That's where there's a local minimum.

Clarifier: Wait. The first derivative is indeed 0 there, but you have to check the second derivative, too. I did, and it's also 0 when $x = 3$. This means that f has an inflection point there, so there's neither a local maximum nor a local minimum when $x = 3$.

Reporter: Well, I'd like to report that I just graphed f, and it has a local **maximum** when $x = 3$!

Assume that the manager and clarifier did do the derivative computations correctly. Could the reporter's claim be correct? Explain.

Solution

Answer: The conclusion of the *Reporter* could be correct. The conclusions of the *Manager* and *Clarifier* are not necessarily correct.

What they are doing wrong: The *Manager* is forgetting that you need to test a critical point to see if it is a local maximum, a local minimum or neither (so his conclusion can be wrong). The *Clarifier* makes a good point saying that you need to test your point, and he seems to suggest a test (remember that to apply the second derivative test for local minima and local maxima you check if your second derivative is positive or negative, respectively, but the test tells nothing when it is zero). However the *Clarifier* is making a mistake while testing for a point to be an inflexion point (remember that these are the points where concavity changes). When the second derivative is zero at my point, we identify the change in concavity by looking for a change in the sign of the second derivative occurring at our point. The *Clarifier* is forgetting to check this (so his conclusion can be wrong too).

The *Reporter*'s claim could be correct: There are examples of functions that have a local maximum at $x = 3$ and that have first and second derivative equal to zero at $x = 3$. For instance $f(x) = -(x - 3)^4$ satisfies these conditions. So the *Reporter* could be correct.

(2) Let $f(x)$ be a function that is everywhere differentiable. For the following questions suppose that $\mathbf{f'(x)}$ is continuous and its behavior is as the table below suggests that you know the values for $f'(x)$ given in the table below.

$x =$	-3	-2	-1	0	1
$\mathbf{f'(x)} =$	2	0.5	-0.5	-1	-0.5

(a) Identify the location of any critical points and local maxima or local minima, if any, that this data indicates $f(x)$ has.

(b) If possible, identify the location of any inflection points of $f(x)$, and the concavity of the graph of $f(x)$. If it is not possible, briefly explain why.

Solution

(a) Answer: *Critical points:* Some point between $x = -2$ and $x = -1$. *Local Minima:* $x = -3$ and $x = 1$. *Local Maxima:* the critical point.

Explanation: We know that $f(x)$ has critical points at points where $f'(x)$ is zero or undefined. Because $f(x)$ is everywhere differentiable there are no undefined points, and given that $f'(-2) = 0.5$ and $f'(-1) = -0.5$ we know that there is a point between $x = -2$ and $x = -1$ where $f'(x) = 0$. So there is a critical point between these two x-values. Because the sign of $f'(x)$ changes from positive to negative there, we know that this is a local maximum. The function $f(x)$ grows for points to the right and near to of $x = -3$, so $x = -3$ is a local minimum. The function $f(x)$ decreases for points to the left and near to of $x = 1$, so $x = 1$ is a local minimum.

(b) Answer: *Inflection points:* Some point between -1 and 1.

Explanation: Inflection points occur where $f(x)$ changes concavity, which is where $f'(x)$ has a local maximum or minimum. From the data it is clear that $f'(x)$ has a local minimum between $x = -1$ and $x = 1$, so there is an inflection point in the graph of $f(x)$ between those two x-values. Because $f''(x)$ is negative where $f'(x)$ is decreasing, we know that $f(x)$ is concave down before the inflection point and concave up thereafter.