University of Michigan
Student Name:

Math 115 Quiz 7

Section 9 - Winter 2011
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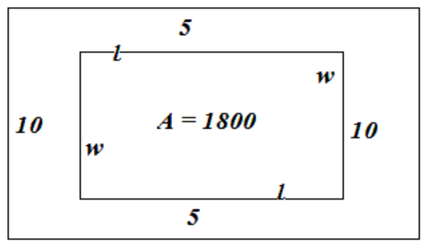
Optimization Problem

A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot wide decks along either side and 10-foot wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.



Solution

Let l and w be the length and width of the pool in feet. We want to minimize the total area A of the property including the deck, which is

$$A = (l + 20) \cdot (w + 10).$$

The area of the pool is 1800 square feet and can be computed also like $l \cdot w$. Then $l \cdot w = 1800$, and therefore $l = \frac{1800}{w}$.

We substitute this into our equation for A to get a relation that involves just one variable. We get:

$$A = (\frac{1800}{w} + 20) \cdot (w + 10) = 1800 + \frac{18000}{w} + 20w + 200 = \frac{18000}{w} + 20w + 2000$$

Our problem is to find the minimum of this expression for $w > 0$.

Then we compute $\frac{dA}{dw}$ to find the critical points:

$$\frac{dA}{dw} = -\frac{18000}{w^2} + 20$$

We see that $\frac{dA}{dw}$ is defined everywhere in $(0, \infty)$. This derivative is zero when $-\frac{18000}{w} + 20 = 0$, that is when $w^2 = 900$, which in $(0, \infty)$ happens just at $w = 30$.

Now we need to analyze our computations to make the conclusion. Over the interval $(0, \infty)$, the derivative $\frac{dA}{dw}$ can just be zero at $w = 30$, and we see from its equation that it is negative for $w < 30$ and positive for $w > 30$. This means that A is decreasing for $w < 30$ and increasing for $w < 30$. Therefore $w = 30$ feet produces the minimum possible area of the property to build the desired pool.

If we substitute in $l = \frac{1800}{w}$, we find that the corresponding value of l is 60 feet.