Section 4.6: Rates and Related Rates

Melvin is in his illegal liquor distillery when he suddenly detects that DEA agent Cyndy is 500 ft directly southwest of him. Melvin's truck is directly south of the distillery and directly east of Cyndy. At the moment when Melvin detects Cyndy's presence, he moves toward his truck at a rate of 3 ft per minute. Also at this moment Cyndy heads toward the truck at 5 ft per minute.

At this moment, how fast is the distance between Melvin and Cyndy shrinking?

Solution



The situation is shown above. We can write an equation involving a, b and c, the lengths of the sides of the triangle, easily using the Pythagorean theorem: $a^2 + b^2 = c^2$. Differentiating both sides of this, we get $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$. We want to find $\frac{dc}{dt}$, the rate at which the distance between them is changing. When c = 500, $a = b = \frac{500}{\sqrt{2}}$ (because the triangle is, initially, isosceles), and we know that $\frac{da}{dt} = -5$ (negative because the distance is decreasing) and $\frac{db}{dt} = -3$. Thus, solving for $\frac{dc}{dt}$, we get $\frac{dc}{dt} = \frac{-8}{\sqrt{2}} \approx -5.7$ ft/min.