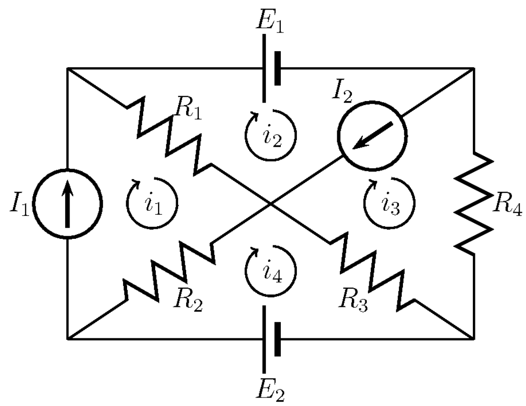
In the *resistor networks* homework problems you will get something like this:



Here the components marked E_1 and E_2 are called *power sources* or *voltage sources*. The components marked R_1, R_2, R_3 and R_4 are called *resistors*. The components marked I_1 and I_2 are called *current sources*.

There is an electric *current* (denoted I) circulating through the circuit which has different values on different branches of the circuit, and it is measured in the unit called *Ampere* (denoted A). The *voltage* (denoted V) is an electric potential that may drop or increase when you pass through one of the components of the circuit, it is measured in Volts (denoted V). The *resistance* of a resistor (denoted R) measured in Ohms (denoted Ω) is related to the voltage drop when you pass through that resistor: This follows *Ohm's Law* "$V = IR$", in words the voltage drop at a resistor is equal to the current through the resistor times the resistance of that resistor. Warning: the voltage and the current are not just given by their value but also by their direction, for example: a voltage drop of 5V in one direction is a voltage drop of -5V in the opposite direction, and a current of 6A in one direction is a current of -6A in the other direction.

The resistor network problems are introduced to practice solving equations. So, *what are the given quantities and what are the unknowns*?

Given Data: You will be given the values of the resistance of all resistors (say $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$ and $R_4 = 4\Omega$), the voltage drops across all power sources (say $E_1 = 1V$ and $E_2 = 2V$, in the direction from the "long leg" to the "short leg"), and the currents through all current sources (say $I_1 = 1A$ and $I_2 = 2A$, in the direction of the arrows).

Basic Questions: Find current through each resistor. Find current through each power source. Find voltage drops across each power source.

Unknowns: Warning: To answer the *basic questions* you do not use what they are asking for as the unknowns!. It is easier to use as unknowns V_1, V_2, i_1, i_2, i_3 and i_4 as we now explain. The first unknowns called V_1 and V_2 are the voltage increments across the current sources I_1 and I_2 (measured in the same direction as the arrows in the current sources). The other unknowns are some "artificial quantities" called *loop currents*. In the picture the loop currents are i_1, i_2, i_3 and i_4 (their direction is shown by an arrow). The strategy is to write everything using these loop currents, then we solve for them, and finally we answer the questions.

Equations: You write *one equation for each current source* and *one equation for each loop*. In the example above you get six equations (one for each of I_1 and I_2, and one for each of the four loops with the marks i_1, i_2, i_3 and i_4). The equation of a current source says that the total current that enters is exactly the current that leaves. The equation of a loop says that if you go around that loop one time and add the voltage drops produced by each component you obtain zero. Warning: You need to be careful with the signs (you can go around a loop clockwise or counterclockwise, but need to mind the signs of the quantities).

Let's write the equations for the example above:

For current Source I_1: The currents going upwards through I_1 are (on one hand) in terms of loop currents i_1 and (on the other hand) in terms of the given data I_1 (which is 1A). So we get $\boxed{i_1 = 1}$.

For current Source I_2: Similarly, from north-east direction to south-west direction, on the one hand in terms of loop currents you have $i_2 - i_3$, on the other hand in terms of the given data this is I_2 (which is 2A). Making these equal we get $\boxed{i_2 - i_3 = 2}$.

For the First Loop: Starting at the north-west corner and going clockwise, first we encounter a resistor. Using Ohm's law the voltage drops there by $V = I \cdot R = (i_1 - i_2) \cdot 1$. Then we encounter a second resistor. Using Ohm's law the voltage drops there by $V = I \cdot R = (i_1 - i_4) \cdot 2$. Then we find a current source. The voltage drops there by $-V_1$. These have to add to zero so we get the equation $(i_1 - i_2) + 2(i_1 - i_4) - V_1 = 0$, which simplifies to $\boxed{3i_1 - i_2 - 2i_4 - V_1 = 0}$.

For the Second Loop: Starting at the north-west corner and going clockwise, first we encounter a power source. The voltage drop there is E_1 (not $-E_1$, watch out!) which is 1V. Then we encounter a current source. The voltage drop there is $-V_2$. Then we find a resistor. Using Ohm's law the voltage drops there by $V = I \cdot R = (i_2 - i_1) \cdot 1$. These have to add to zero so we get the equation $1 - V_2 + (i_2 - i_1) = 0$, which simplifies to $\boxed{-i_1 + i_2 - V_2 = -1}$.

For the Third Loop: Proceed similarly and you get $\boxed{7i_3 - 3i_4 + V_2 = 0}$.

For the Fourth Loop: Proceed similarly and you get $\boxed{-2i_1 - 3i_3 + 5i_4 = 2}$.

Now we have six equations on six unknowns and we proceed to solve them. I got:

$$\boxed{i_1 = 1A,\ i_2 = \tfrac{64}{31}A,\ i_3 = \tfrac{2}{31}A,\ i_4 = \tfrac{26}{31}A,\ V_1 = -\tfrac{23}{31}V,\ V_2 = \tfrac{64}{31}V}$$

Now all questions that they ask you in the homework can be answered directly from what we did. Let us see examples:

• Find the loop current i_4. *Solution:* We did this! $i_4 = \frac{26}{31}A$.

• Find the current through resistor R_3 going from south-east to north-west. *Solution:* Easy! it is $i_3 - i_4 = \frac{2}{31}A - \frac{26}{31}A = -\frac{24}{31}A$ (so the current there actually flows in the other direction!).

• Find the voltage drop across resistor R_3 going from south-east to north-west. *Solution:* Easy! By Ohm's law it is $V = IR = (-\frac{24}{31})(3) = -\frac{72}{31}V$.

• Use Kirchhoff's Voltage Law to find an equation for the voltage drop around the first loop. *Solution:* Easy! We did this above! it is just a fancy way of asking for the equation that we got above for the first loop $\boxed{3i_1 - i_2 - 2i_4 - V_1 = 0}$ (what we used there is called *Kirchhoff's Voltage Law*).

• Are the cells E_1 and E_2 being charged or discharged? *Solution:* Easy! We just look at the sign of the current through each cell to see if the current enters or leaves from the cell. Since $i_2 = \frac{64}{31}A$ is positive we see from the graph that the current flows to E_2 so it is being charged. Since $i_4 = \frac{26}{31}A$ is positive we see from the graph that the current flows from E_2 so it is being discharged.

• Suppose that I_1 is adjustable so that it is no longer 1A. What should I_1 be changed so that there is no current through cell E_1? *Solution:* This problem requires more work! They completely changed the problem! We need to find a new system of equations. The equation $i_2 = 0$ is introduced (so that no current goes through E_1). The old first equation disappears completely because now the current $I_1 = i_1$ (going upwards) is what we want to find. The other five equations from before stay as they are. We solve the new system (six equations, with six unknowns) and find that $i_1 = -\frac{58}{11}$. So the value that I_1 needs to be changed to is $I_1 = i_1 = -\frac{58}{11}$ (to answer this question we did not need to solve for any the other unknowns i_2, i_3, i_4, V_1 and V_2).

• (In the original problem) Use Kirchhoff's voltage law to find an equation around the outer loop. *Solution:* Easy! We go clockwise starting from the north-west corner. First we find a power source where the voltage drops by $E_1 = 1V$. Then we find a resistor, where by Ohm's law the voltage drops by $V = IR = 4i_3$. Then we find a power source where the voltage drops by $-E_2 = -2V$. Then we find a current source where the voltage drops by $-V_2$. These add to zero, so we get $1 + 4i_3 - 2 - V_2 = 0$, which simplifies to $\boxed{4i_3 - V_2 = 1}$.