University of British Columbia
Student Name and Number:

Math 152 Quiz 1

Section 202 - Winter 2013
Instructor: Jose Gonzalez
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INSTRUCTIONS (Read carefully): Please answer the **two** questions below. **Only the front side of the page is marked** (so you can use the back side find the solution and to check your work, but the full solution with the procedure must be at the front page to get marks). To get credit you need to **show all your work**.

The three points $u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ lie on a plane in 3D space with coordinates x, y and z.

(1) Find a description of this plane in **parametric form**.

Solution To provide a parametric description of the plane we first find a point in the plane and two nonparallel vectors that are parallel to the plane.

The plane passes through the end point of the vector u. The plane contains the vector that goes from the end point of u to the end point of v and the vector that goes from the end point of u to the end point of w. Those two vectors are equal to $v-u$ and $w-u$ respectively, and they serve as the vectors that are parallel the plane (and not parallel to each other).

Then, a parametric description of this plane is $u + t\,(v-u) + s\,(w-u)$ where t and s are free parameters, that is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + s \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}, \quad \text{where } t \text{ and } s \text{ are free parameters}$$

(2) Find a description of this plane in **equation form**.

Solution The equation description has the form $ax + by + cz = d$. We can find a, b and c using that $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is a vector perpendicular to the plane. To obtain a vector perpendicular to the plane we do the cross product of two vectors (non parallel to each other) which are contained in the plane. For instance, we can use the vectors $v-u$ and $w-u$ (used also in **(1)**) which are contained in the plane. We have:

$$(v-u) \times (w-u) = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} = \det \begin{bmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

Then we can use an equation of the form $4x + 2y + 2z = d$. To find d, we use that the point u satisfies the equation of the plane. We have, $4 \cdot 1 + 2 \cdot 2 + 2 \cdot 2 = d$, and then $d = 8$.

Therefore, a description of the plane in equation form is

$$4x + 2y + 2z = 8.$$