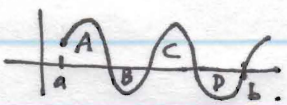
Math 101: Review Midterm 1.

▷ Definite & Indefinite Integral

- Definite: (limit of Riemann-Sums)

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{n=1}^{\infty} f(x_i)\,\Delta x = A - B + C - D.$$

Intuitively: Signed area



- Indefinite: Antiderivative.

$$\int 2x\,dx = x^2 + C$$

$$\int \ln x = x\cdot\ln x - x + C.$$

▷ Fundamental Theorem of Calculus:

- How to take derivative of an integral:

$$F(x) = \int_7^x f(t)\,dt \Rightarrow F'(x) = f(x).$$

Example/Exercise: $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt = f(g(x))\cdot f'(x) - f(h(x))\cdot h'(x).$

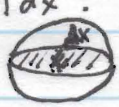
- How to compute definite using indefinite (antiderivative):

If want $\int_a^b f(x)\,dx$ and have $\int f(x)\,dx = F(x) + C \Rightarrow \int_a^b f(x)\,dx = F(b) - F(a).$

▷ Applications:

- Average value of fns: $\text{Avg} = \frac{1}{b-a}\int_a^b f(x)\,dx.$

- Areas: $A = \int_a^b |f(x) - g(x)|\,dx.$
- Volumes: $V = \int_a^b A_x\,dx$



$$V = \int_a^b \left(\pi f^2(x) - \pi g^2(x)\right)dx$$

- Work: $W = \text{Force}\cdot\text{distance}.$

When Force changes divide movement into tiny movements and integrate
When distance changes divide object into slabs and integrate.

▷ Techniques to compute integrals.

(we focus on Indefinite because then FTC gives you the Definite as well).

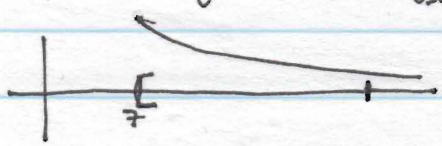
- Substitution: $u = f(x)$, $du = f'(x)\,dx$ substitute, integrate, get back to x!!
- By parts: $\int u\,dv = uv - \int v\,du$. Suggestion: Choose u & dv according to ULIATEV.
- Partial Fractions: for $\frac{\text{Polynomial}}{\text{polynomial}}$.
- Trig integrals & trig substitutions:
  - $x^2 + a^2$: $x = a\tan\theta$, $dx = a\cdot\sec^2\theta$ $\quad(\tan^2\theta + 1 = \sec^2\theta)$
  - $x^2 - a^2$: $x = a\sec\theta$, $dx = a\cdot\sec\theta\cdot\tan\theta$
  - $a^2 - x^2$: $x = a\cdot\sin\theta$, $dx = a\cdot\cos\theta$ $\quad(\sin^2\theta + \cos^2\theta = 1)$

e.g. $\int\sqrt{100 - 81x^2}\,dx = \int 9\sqrt{\left(\frac{10}{9}\right)^2 - x^2}\,dx$

use $x = \frac{10}{9}\cdot\sin\theta$.

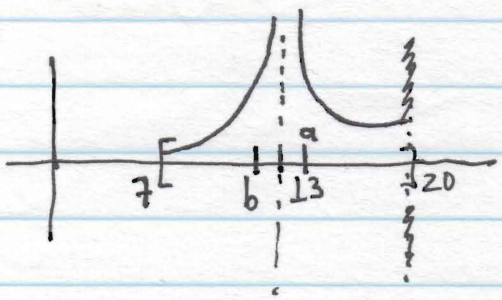
▷ Improper Integrals: Allow infinite intervals & vertical asymptotes by using limits.

- Infinite Interval



$$\int_7^\infty f(x)\,dx = \lim_{t\to\infty}\int_7^t f(x)\,dx.$$

- Vertical Asymptote.



$$\int_7^{20} f(x)\,dx = \lim_{b\to 13^-}\int_7^b f(x)\,dx + \lim_{a\to 13^+}\int_a^{20} f(x)\,dx.$$