University of British Columbia
Student Name:

Math 101 Quiz 3

Section 202 - Winter 2014
Instructor: Jose Gonzalez
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Instructions: Please solve the exercises below. To get any credit you must provide a complete explanation.

(1) (*5 points*) Determine if the following series converges or diverges: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(2) (*5 points*) Determine the exact value of the series: $\displaystyle\sum_{n=0}^{\infty} \frac{4 \cdot 2^n}{3 \cdot 3^n}$

(3) (*5 points*) Determine if the following series converges or diverges: $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$

Solution (1)

We have that $n^2 \leq n^2+1$ for all n, so $0 \leq \frac{1}{n^2+1} \leq \frac{1}{n^2}$ for all n. Since the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges as it is a p-series with $p = 2 > 1$, then by **comparison test** the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges as well.

Solution (2)

The given series is a **geometric series** with ratio $r = \frac{2}{3}$, and since $|r| = \frac{2}{3} < 1$ it converges. Its initial value is $a = \frac{4}{3}$. Then the value of the series is $\displaystyle\sum_{n=0}^{\infty} \frac{4 \cdot 2^n}{3 \cdot 3^n} = \frac{a}{1-r} = \frac{4/3}{1-2/3} = 4.$

Solution (3)

Let $f(x) = \frac{1}{x \cdot \ln(x)}$. Since $x \cdot \ln(x)$ is positive, continuous and increasing for $x > 1$, then $f(x)$ is positive, continuous and decreasing for $x > 1$, so we can apply **integral test**. We compute the corresponding improper integral:

$$\int_2^{\infty} f(x)\,dx = \int_2^{\infty} \frac{1}{x \cdot \ln(x)}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{1}{x \cdot \ln(x)}\,dx = \lim_{t\to\infty} ln(ln(x))|_2^t = \lim_{t\to\infty} (ln(ln(t)) - ln(ln(2))) = +\infty.$$

The improper integral diverges, so by the integral test the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$ diverges as well.