Math 10A
Midterm 1
Review

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1. Know how to find the equation of a line (preferably parametric equations):
   (a) Containing two points. (1.1)
   (b) Containing a point and generated by a vector. (1.1)
   (c) Perpendicular to a plane. (1.2)
2. Know how to find expressions representing planes:
   (a) The set of points of the form $sv_1 + tv_2$ where s and t are real numbers and v_1 and v_2 are vectors. (1.1)
   (b) The equation of a plane containing three points. (1.3)
   (c) The equation of a plane containing a point and perpendicular to a line. (1.3)
   (d) The equation of a plane containing two parallel lines. (1.3)
   (e) The equation of a plane spanned by two vectors. (1.3)
3. Computations in $\mathbb{R}^n$.
   (a) Find the dot product of any two vectors. (1.2)
   (b) Find the length of a vector. (1.2)
   (c) Find the projection of a vector onto another vector. (1.2)
   (d) Find the angle between two vectors. (1.2)
   (e) Verify Cauchy-Schwarz Inequality or Triangle Inequality for any two vectors. (1.2)
4. Cylindrical and Spherical Coordinates.
   (a) Conversion between Cartesian, Cylindrical and Spherical Coordinates. (1.4)
   (b) Describe the set of points when one of the variables is constant in any of the coordinate systems. (1.4)

5. Computations in $\mathbb{R}^3$.

   (a) Find the cross product between two vectors. (1.3)

   (b) Find the area of a parallelogram. (1.3)

   (c) Find the distance from a point to a plane. (1.3)

   (d) Find the intersection of two planes (if any). (1.3)

   (e) Find the intersection of a line and a plane (if any). (1.3)

   (f) Find the intersection of two lines (if any). (1.3)

6. Matrices.

   (a) Find the sum of any two $n \times m$ matrices. (1.5)

   (b) Find the product of a $n \times m$ matrix and a $m \times k$ matrix. (1.5)

   (c) Find the determinant of and 2×2, 3×3 matrix or an $n \times n$ matrix for $n \geq 4$ with ample 0's. (1.5)

   (d) Know the determinant rules $|AB| = |A||B|$ and $|kA| = k^n|A|$. (1.5)

7. Vector valued functions:

   (a) Find the domain of a vector valued function. (2.1)

   (b) Find the range of a vector valued function. (2.1)

   (c) Find and graph the level sets of a real valued function. (2.1)

8. Know how to find limits of functions in 2 or more variables:

   (a) Use polar coordinates to evaluate functions in 2 variables when applicable. (2.2)

   (b) Use spherical or cylindrical coordinates to evaluate functions in 3 variables when applicable. (2.2)

   (c) Use limits from 1 variable calculus such as $\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, etc. to help determine limits for functions of 2 or more variables. (2.2)

   (d) Use different lines going through point to show a limit does not exist. (2.2)

9. Find 1st and 2nd partial derivatives of functions of 2 or more variables. (2.3 and 3.1)

10. Find the tangent vector/tangent line to a point on a curve (parametric derivatives). (2.4)

11. Find the matrix of derivatives of a vector valued function, in particular find the gradient. (2.3)

12. Know how to apply the chain rule, i.e. use matrix multiplication. (2.5)

13. Find directional derivatives, i.e. $\nabla f \cdot h$. (2.6)

14. Find the tangent plane at a point for a function of two variables or for a surface defined by three variables. (2.3 and 2.6)

15. Find the 1st and 2nd order Taylor polynomial for a function in 2 or more variables. (3.2)

16. Find critical values. Identify them as local maxima, local minima or saddle point. (3.3)

17. Find absolute extrema of a function using lagrange multipliers on the boundary. (3.3 and 3.4)

18. Find the extrema of a function restricted to a set using lagrange multipliers. (3.4)

19. Use the implicit or inverse function theorem to show that locally some of the variables may be solved in terms of the others. (3.5)

20. Find the derivatives of implicitly defined functions. (3.5)

21. Find the acceleration and force of a particle traveling on a curve at a point. (4.1)

22. Find the arclength of a curve. (4.2)

23. Verify that a curve is a flow line for a vector field. (4.3)

24. Compute the divergence and the curl of a vector field. (4.4)