Homework 1  
MATH 120  
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1. Find the transformation matrix $T$ from the basis $B = \{(1, 0, 0)^T, (1, 1, 0)^T, (1, 1, 1)^T\}$ to the standard basis.

2. Give the matrix representing the linear transformation

$$L(x, y, z) = (2x - 3y, x + 4z, 2y - 5z)$$

with respect to the standard basis.

3. Give the matrix representing the linear transformation

$$L(x, y, z) = (2x - 3y, x + 4z, 2y - 5z)$$

with respect to the basis $(1, 0, 0)^T, (1, 1, 0)^T, (1, 1, 1)^T$.

4. Find the eigenvalues and their associated eigenvectors of the matrix

$$\begin{pmatrix}
1 & 2 & -1 \\
1 & 0 & 1 \\
4 & -4 & 5
\end{pmatrix}.$$ 

5. Find an orthogonal matrix which diagonalizes

$$\begin{pmatrix}
1 & 2 \\
2 & 1
\end{pmatrix}.$$ 

6. Find the orthogonal complement of the subspace $W = \text{span}\{(1, 2, 0, 1)^T, (1, 0, 3, 2)^T\}$.

7. Find a symmetric matrix which represents the quadratic form

$$f(x, y, z) = x^2 + 4xy - xz + 6yz - 3z^2.$$ 

8. Determine if the matrix

$$\begin{pmatrix}
1 & 2 & -1 \\
2 & 5 & 3 \\
-1 & 3 & 5
\end{pmatrix}$$

is positive definite, negative definite or neither.

9. Problem 3.5 on page 35.