

# Homework

## MATH 131

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1. Determine if  $\mathbb{R}^2$  is a vector space with addition defined normally, i.e.  $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$  and scalar multiplication defined as  $r \circ (x, y) = (rx, y)$ .
2. Determine if  $\mathbb{Z}$  is a vector space with addition defined in the usual way and scalar multiplication defined as  $r \circ n = \lfloor r \rfloor n$  where  $\lfloor \cdot \rfloor$  is the greatest integer function.
3. Determine if  $\mathbb{R}$  is a vector space with addition defined as  $x \dagger y = x + y - 1$  and scalar multiplication defined as  $r \circ x = rx - r + 1$ .
4. If  $\mathbf{x}, \mathbf{y}, \mathbf{z}$  are all vectors in a vector space  $V$ . Show if  $\mathbf{x} + \mathbf{y} = \mathbf{x} + \mathbf{z}$ , then  $\mathbf{y} = \mathbf{z}$ .
5. Show that additive inverses are unique.

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6. Decide whether the subset of polynomials of degree 3 is a subspace of the vector space of polynomials of degree 3 or less.
7. Decide if the set of constant functions is a subspace of the vector space of all real valued functions.
8. Decide if the set  $\{\sec^2 x, \tan^2 x, 4\}$  is linearly independent in the vector space of functions.
9. Decide if the set  $\{x^2 + 1, x - 1, x + 1\}$  is a basis for the vector space of polynomials of degree three or less.
10. Suppose  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$  spans a vector space  $V$ .
  - (a) If we add  $\mathbf{w}$  will  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n, \mathbf{w}$  still span  $V$ ?
  - (b) If we take away one of the  $v_i$ , will  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n \setminus \mathbf{v}_i$  still span  $V$ ?
11. Let  $U$  and  $W$  be subspace of a vector space  $V$ . Show  $U + W = \{z | z = u + w \text{ where } u \in U \text{ and } w \in W\}$  is a subspace of  $V$ .
12. If  $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$  are a linearly independent set of vectors in a vector space  $V$ , show that  $\mathbf{v}_2, \dots, \mathbf{v}_n$  cannot span  $V$ .