Homework
MATH 131

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1. Find a matrix representing the identity map on $\mathbb{R}^n$ from the basis $B = \{(1,2),(2,5)\}$ to the standard basis e_1, e_2.

2. Find a matrix representing the map $T(x,y,z) = (2x, 3y, 5z)$ from the standard basis to the basis $B = \{(3,0,1),(0,1,2),(1,4,3)\}$

3. Let $b_1 = (1,0,1), b_2 = (1,1,0), b_3 = (0,1,1)$. Let $L : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $L(x_1,x_2) = x_1 b_1 + x_2 b_2 + (x_1 + x_2) b_3$. Find a matrix A representing L with respect to the standard basis $\{e_1, e_2\}$ and $\{b_1, b_2, b_3\}$.

4. Let $L : P_3 \to P_2$ be the linear transformation defined by $L(p(x)) = p'(x) + p(0)$. Find the matrix representation for L with respect to the ordered bases $S = \{x^2, x, 1\}$ and $B = \{2, 1 - x\}$. Then find coordinate vector $L(x^2 + 2x - 3)_B$.

5. Let $E = \{(1,0,-1),(1,2,1),(-1,1,1)\}$ and $F = \{(1,-1),(2,-1)\}$. Find the matrix representing $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $L(x_1,x_2,x_3) = (x_1 + x_2, x_1 - x_3)$ with respect to the bases E and F.

6. Let V and W be vector spaces with finite bases E and F respectively. If $L : V \to W$ is a linear transformation and A is the matrix defining L with respect to the bases E and F, show that $v \in ker(L)$ if and only if $[v]_E \in N(A)$.

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7. Determine if $< (x_1,y_1),(x_2,y_2) >= 2x_1x_2 + 3y_1y_2$ defines an inner product on $\mathbb{R}^2$.

8. Define the norm $< p(x), q(x) >= \int_0^1 p(x)q(x)dx$ for the vector space P of polynomials. Show that the vectors x and x^2 are not orthogonal.

9. Suppose $u, v \in V$ such that $\|u\| = 3, \|u + v\| = 4, \|u - v\| = 6$. What is $\|v\|$?

10. Prove for any inner product that $< u, v >= \dfrac{\|u+v\|^2 - \|u-v\|^2}{4}$.