Homework 9  
MATH 132  
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1. Determine the conjugate transpose of the matrix
   \[
   \begin{pmatrix}
   1 + i & 2i & 3 - i \\
   2 + i & 4 & -1 + 2i \\
   2 & 4 + i & -5i 
   \end{pmatrix}
   \]

2. What type of matrix is \[
   \begin{pmatrix}
   3 & 2 - i \\
   2 + i & 4 
   \end{pmatrix}
   \]

3. What type of matrix is \[
   \begin{pmatrix}
   \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}} \\
   \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i 
   \end{pmatrix}
   \]

4. A matrix \(A\) with complex coefficients is skew Hermitian if \(A^* = -A\). If \(M\) and \(N\) are \(n \times n\) real matrices. Show \(A = M + iN\) is skew Hermitian if and only if \(M\) is skew symmetric and \(N\) is symmetric.

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5. Find a unitary matrix \(U\) which diagonalizes \[
   \begin{pmatrix}
   2 & -i \\
   i & 0 
   \end{pmatrix}
   \]

6. Find a unitary matrix \(U\) which diagonalizes \[
   \begin{pmatrix}
   0 & 1 - i & 4 \\
   1 + i & 0 & 0 \\
   4 & 0 & 1 
   \end{pmatrix}
   \]

7. Show that every skew Hermitian matrix is unitarily diagonalizable.

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8. If \(A\) is a normal matrix, show that the column space of \(A\) is the column space of \(A^*\).

9. Suppose \(A\) is a normal matrix satisfying \(A^9 = A^8\). Show that \(A\) is Hermitian and that \(A^2 = A\).

10. Show that every normal matrix \(A\) has a square root, i.e. there is a matrix \(B\) such that \(B^2 = A\).