

Practice Midterm 1

MATH 9C

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1. Determine if the following improper integral converges or diverges:

$$\int_1^e \frac{1}{x\sqrt{\ln x}} dx$$

2. Consider the sequence $a_n = \frac{1 + (-1)^n}{n}$.

- (a) Is a_n bounded below? If so, find a lower bound.
- (b) Is a_n non increasing? Explain your answer.
- (c) Find $\lim_{n \rightarrow \infty} a_n$.

3. List the first three partial sums of $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$.

4. Suppose a_n and b_n are sequences and $\sum_{n=1}^{\infty} a_n + b_n$ converges. Is it true that $\sum_{n=1}^{\infty} a_n$ converges? Why or why not?

5. Find the sum of the series $\sum_{n=0}^{\infty} \frac{2(-1)^n}{5^n}$

6. Suppose a_n is a sequence which is negative, increasing and $a_n = f(n)$ for $n \geq 1$ for some continuous function $f(x)$ on $[1, \infty)$. How might one determine if $\sum_{n=1}^{\infty} a_n$ is convergent or not?

7. Determine if the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2-n+1}$

(b) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

(c) $\sum_{n=1}^{\infty} \frac{5}{n^n}$

(d) $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(f) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

8. Determine if the following series are conditionally convergent, absolutely convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(c) $\sum_{n=1}^{\infty} \frac{(2n)^{2n}}{(2n^2+1)^n}$