# Practice Midterm 2
# MATH 9C

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1. Determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}(x+2)^n$

2. Determine the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2^n}{n!}(3x-1)^n$

3. If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has a radius of convergence equal to 5, does $f(5)$ necessarily converge?

4. Find the 3rd Taylor polynomial for $f(x) = \frac{1}{x}$ centered at $x = 2$.

5. Find the 2nd Taylor polynomial for $f(x) = \sin(2x)$ centered at $x = \frac{\pi}{3}$.

6. Find the Taylor series for $f(x) = \ln x$ centered at $x = 1$.

7. Find the Maclaurin series for $f(x) = \dfrac{\sin(2x)}{x}$.

8. Find the Maclaurin series for $f(x) = e^{x^2}$.

9. Show that $y = xe^x$ is a solution to the differential equation $y' = \dfrac{(x+1)y}{x}$.

10. Find the general solution to $y' = (y^2+1)\sec^2(x)$.

11. Find the solution to the initial value problem $xy' - y = x^2 e^x$, $y(0) = 2$.

12. Find the general solution to $y' - 4y = e^{3x}$.

13. A 20 gallon container contains 10 gallons of water and one half a pound of salt. Pure water is poured into the container at a rate of 2 gallons a minute. If the subsequent mixture leaves the container at a rate of 1 gallon a minute, set up a differential equation with initial conditions to solve for the amount of salt in the container at any time $0 < t < 5$.

14. A population of bacteria grows at a rate proportional to the amount present. If the population doubles in two hours, find the general solution for the amount of bacteria present.

15. A bicyclist is coasting down the road at a rate of 3 meters per second. If the bicyclist and his bicycle weigh 70 kg and the resistance is given by twice the speed he is traveling, find his velocity after 1 minute.

16. Find the series solution to $y'' - y = 0$ with initial conditions $y'(0) = 0$ and $y(0) = 1$.

17. Use series to evaluate $\lim_{x \to 0} \frac{\ln(1+x)}{\sin x}$.