

Practice Midterm 1

MATH 9B

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1. Write a lower sum to approximate the area under $y = 4 - x^2$ on the interval $[0, 2]$ with $n = 4$ equal subintervals.

$$(4 - \frac{1}{4})\frac{1}{2} + (4 - 1)\frac{1}{2} + (4 - \frac{9}{4})\frac{1}{2} + (4 - 4)\frac{1}{2} = \frac{17}{4}$$

2. You and a companion are driving in a car with a broken odometer; however, the speedometer works. Your friend records the speed at 1 minute intervals for 6 minutes. The table below gives the speeds.

Time (in minutes)	Speed (in miles per hour)
0	25
1	35
2	20
3	40
4	35
5	30
6	60

What is an upper estimate for the distance you and your friend have traveled in that time?

An upper sum will give an upper estimate for the distance traveled. Note a minute is $1/60$ of an hour.

$$\text{Upper Sum} = 35(1/60) + 35(1/60) + 40(1/60) + 40(1/60) + 35(1/60) + 60(1/60) = 275/60 = 4\frac{7}{12} \text{ miles.}$$

3. Express $\frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5}$ using sigma notation.

$$\frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5} = \sum_{i=1}^4 \frac{i^2}{5}$$

4. Write $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \sqrt{c_k} \Delta x_k$, where P is a partition of $[1, 9]$, as a definite integral.

$$\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \sqrt{c_k} \Delta x_k = \int_1^9 \sqrt{x} dx$$

5. Find the area under $f(x) = 6 - 5x - x^2$ on the interval $[0, 1]$

$$A = \int_0^1 6 - 5x - x^2 dx = 6x - \frac{5x^2}{2} - \frac{x^3}{3} \Big|_0^1 = 6 - \frac{5}{2} - \frac{1}{3} = \frac{19}{6}$$

6. Find $\frac{d}{dx} \int_0^{x^2} \frac{dt}{1 - \cos t}$

$$\frac{d}{dx} \int_0^{x^2} \frac{dt}{1 - \cos t} = \frac{2x}{1 - \cos x^2}$$

7. Find the average value of $f(x) = 1 - \sin x$ on $[0, \frac{\pi}{2}]$

$$\text{avg}(f) = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} 1 - \sin x dx = \frac{2}{\pi} (x + \cos x) \Big|_0^{\frac{\pi}{2}} = \frac{2}{\pi} (\frac{\pi}{2} - 1) = 1 - \frac{2}{\pi}$$

8. Find the area between $f(y) = \sec^2 y$ and $g(y) = \sin y$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 y - \sin y dy = \tan y + \cos y \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 1 + \frac{\sqrt{2}}{2} - (-1 + \frac{\sqrt{2}}{2}) = 2$$

9. Find the area enclosed by $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$.

$$A = \int_{-1}^0 x^3 - \sqrt[3]{x} dx + \int_0^1 \sqrt[3]{x} - x^3 dx = \frac{x^4}{4} - \frac{3x^{\frac{4}{3}}}{4} \Big|_{-1}^0 + \frac{3x^{\frac{4}{3}}}{4} - \frac{x^4}{4} \Big|_0^1 = 0 - (\frac{1}{4} - \frac{3}{4}) + \frac{3}{4} - \frac{1}{4} - 0 = 1$$

10. Evaluate $\int_0^2 x(4 - x^2)^5 dx$

Let $u = 4 - x^2$, then $du = -2x dx$ and $u(0) = 4$ and $u(2) = 0$. Thus $\int_0^2 x(4 - x^2)^5 dx = -\frac{1}{2} \int_4^0 u^5 du = -\frac{1}{2} \cdot \frac{u^6}{6} \Big|_4^0 = 0 + \frac{4^6}{12} = \frac{1024}{3}$

11. Given $\int_0^5 f(x) dx = -5$ and $\int_0^3 f(x) dx = 7$, what is $\int_3^5 -f(x) dx$?

$$\int_3^5 -f(x) dx = \int_0^5 -f(x) dx - \int_0^3 -f(x) dx = 5 + 7 = 12$$

12. Set up the integral to find the volume of the solid given by revolving the region bounded by $y = x^2 - 4x + 2$ and $y = 2$ about the line $y = 2$.

To find the limits of integration, set $2 = x^2 - 4x + 2$. So $x^2 - 4x = 0$ which implies $x = 0, 4$. On this region $y = 2$ is above $y = x^2 - 4x + 2$. Revolving about $y = 2$ which is a horizontal line, you would need to use discs. The radius of the disc is $r(x) = 2 - (x^2 - 4x + 2) = 4x - x^2$. Hence, the volume will be given by $V = \int_0^4 \pi(4x - x^2)^2 dx$.

13. Set up the integral to find the volume of the solid given by revolving the region bounded by $y = x$ and $y = 6 - x^2$ about the line $x = -3$.

To find the limits of integration set $x = 6 - x^2$ or $x^2 + x - 6 = 0$. Thus $x = -3, 2$. Revolving about the vertical line $x = -3$, one needs to use shells. The radius would be $r(x) = x - (-3) = x + 3$. The height would be $h(x) = 6 - x^2 - x$. Thus $V = \int_{-3}^2 2\pi(x + 3)(6 - x^2 - x) dx$.

14. Set up the integral to find the volume of the solid given by revolving the region bounded by $x = y + 1$, $x = 2$ and the x -axis about the y -axis.

The limits of integration will be $y = 0$ and $y = 1$. As the line is vertical and the function is given in terms of y , the method you need to use is washers. The outer radius is $R(y) = 2$ and the inner radius is $r(y) = y + 1$. Thus $V = \int_0^1 \pi(2^2 - (1+y)^2)dy$.

15. Find the length of the parametric curve $x = \frac{8\sqrt{t^3}}{3}$, $y = (t - 1)^2$ on the interval $[2, 4]$.

To find the length of a parametric curve you need to find $x'(t) = 4t^{\frac{1}{2}}$ and $y'(t) = 2(t - 1)$.

$$s = \int_2^4 \sqrt{16t + 4(t - 1)^2} dt = \int_2^4 \sqrt{4(t + 1)^2} dt = \int_2^4 2(t + 1) dt = t^2 + 2t \Big|_2^4 = 24 - 8 = 16.$$

16. Set up the integrals to find the center of mass for the lamina plate of density 1 given by the region bounded by $y = x^2$ and $y = 2x$.

The limits of integration are $x = 0$ and $x = 2$ obtained by setting the two functions equal to each other. The three integrals are $m = \int_0^2 2x - x^2 dx$, $M_y = \int_0^2 x(2x - x^2) dx$ and $M_x = \int_0^2 \frac{2x + x^2}{2}(2x - x^2) dx$ and $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$.

17. Set up the integral to find the surface area of the sphere of radius 2.

The equation of a circle of radius two is $x^2 + y^2 = 4$. The sphere is obtained by revolving the semicircle given by $y = \sqrt{4 - x^2}$ on $[-2, 2]$ about the x -axis. Thus the surface area is given by $SA = \int_{-2}^2 2\pi\sqrt{4 - x^2} \sqrt{1 + (\frac{-2x}{2\sqrt{4 - x^2}})^2} dx = \int_{-2}^2 2\pi\sqrt{4 - x^2 + x^2} dx = \int_{-2}^2 4\pi dx$.

18. Set up the integral to find the work done in stretching a spring 1 foot if a force of 20 pounds is used to stretch the spring half a foot.

First we need to find the spring constant. $k = F/x = 20/.5 = 40$. So $W = \int_0^1 40x dx$ foot-pounds.

19. How much work is done in filling an empty conical tank from a hole in the bottom with oil of weight 60 lb/ft³ illustrated below.

The height of the cone is 10 feet and the radius is 3 feet. So the radius of each slice is $x = r(y) = \frac{3y}{10}$. So the area of the slice is $A(y) = \frac{9\pi y^2}{100}$ and the weight of each slice is $60 \cdot \frac{9\pi y}{100} dy = \frac{27\pi y^2}{5} dy$. Each slice is displaced y . So the work done is given by $W = \int_0^{10} \frac{27\pi y^3}{5} dy = \frac{27\pi y^4}{20} \Big|_0^{10} = 13500\pi$ foot pounds.