Practice Final
MATH 9A

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1. What are the dimensions in inches of the largest rectangle which can be inscribed inside the ellipse $\frac{x^2}{8} + \frac{y^2}{9} = 1$?

2. The surface area of a sphere is changing at a rate of 25 square millimeters per second. At what rate is the volume of the sphere changing when the radius is 10 millimeters? (Hint: The surface area and volume of a sphere are given by the formulas $SA = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$.)

3. The diameter of a tree was 12 inches. During the following year the circumference increased 1 inch. About how much did the tree's cross sectional area increase?

4. It took 10 seconds for a mercury thermometer to rise from 0^oF to 100^oF in boiling water. Show that somewhere along the way the mercury was rising at the rate of 10^oF/sec.

5. A rocket lifts off the surface of the earth with a constant acceleration of 25 m/sec^2. How fast will the rocket be going after 10 seconds?

6. Find the intervals where the function $f(x) = 3x^2 - 8x^3$ is increasing.

7. Identify the critical values of $f(x) = \dfrac{x}{4 + x^2}$ as either local maxima, local minima or neither using the first derivative test.

8. Find $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^2}$.

9. Find the antiderivative $F(x)$ of $f(x) = x\sqrt{x^2 - 9}$ satisfying $F(5) = 3$.

10. Give a rough sketch of f given that $f(x)$ is increasing on $(-\infty, -1)$ and $(3, 5)$, decreasing on $(-1, 3)$ and $(5, \infty)$, concave up on $(0, 4)$ and concave down on $(-\infty, 0)$ and $(4, \infty)$ and $f(-1) = 2$, $f(0) = 0$, $f(3) = -2$, $f(4) = 1$ and $f(5) = 4$.

11. Find the tangent line to $f(x) = \tan(2x) + (x+1)\cos x$ at the point $(0, 1)$.

12. Using the definition of the limit, show that $\lim_{x \to 2} 7 - 3x = 1$.

13. For what value of a will $f(x) = \begin{cases} x + a, & x > 2; \\ x^2 + 2x - 3, & x \leq 2 \end{cases}$ be continuous.

14. Suppose f and g are continuous functions with $f(4) = 2$, $g(4) = -1$, $f'(4) = \frac{3}{2}$ and $g'(4) = 7$. Compute the following:

    (a) $\left(\frac{f}{g}\right)'(4)$

    (b) $(5f + fg)'(4)$.

15. If $\lim_{x \to 3^+} f(x) = -1$ and $\lim_{x \to -3^+} f(x) = 4$, could $f(x)$ be an odd function? Explain your answer.