

# Practice Final

## MATH 9B

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1. Write a left sum to approximate the area under  $y = x \sin x$  on the interval  $[0, \pi]$  with  $n = 4$  equal subintervals.
2. You and a companion are sailing with a device that records the speed at any given time. Your friend records the speed at 1 minute intervals for 5 minutes. The table below gives the speeds.

Time (in minutes)	Speed (in miles per hour)
0	60
1	55
2	65
3	45
4	50
5	60

What is an lower estimate for the distance you and your friend have traveled in that time?

3. Write  $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n e^{2c_k} \Delta x_k$ , where  $P$  is a partition of  $[0, 2]$ , as a definite integral.
4. Find the area under  $f(x) = \ln x$  on the interval  $[1, 4]$
5. Find  $\frac{d}{dx} \int_0^{\tan^{-1} x} \frac{dt}{1 - \tan t}$
6. Find the average value of  $f(x) = \frac{1}{\sqrt{4-x^2}}$  on  $[0, 1]$
7. Find the area between  $f(y) = \frac{1}{y}$  and  $g(y) = 1 - y^2$  on the interval  $[1, 3]$ .
8. Find the area enclosed by  $f(x) = x^2 - 2x$  and  $g(x) = x + 10$ .
9. Evaluate  $\int_{-1}^1 (2 + 3x)^3 dx$
10. Given  $\int_0^4 f(x) dx = -7$  and  $\int_4^5 f(x) dx = -2$ , what is  $\int_0^5 2f(x) dx$ ?
11. Set up the integral to find the volume of the solid given by revolving the region bounded by  $y = e^x$ , the  $y$ -axis, the  $x$ -axis and  $x = 2$  about the line  $x = 2$ .

12. Set up the integral to find the volume of the solid given by revolving the region bounded by  $y = x$  and  $y = \sqrt{x}$  about the line  $y = 1$ .
13. Set up the integral to find the volume of the solid given by revolving the region bounded by  $x = \tan^{-1} y$ ,  $x = \frac{\pi}{4}$  and the  $x$ -axis about the  $y$ -axis.
14. Find the length of the curve  $y = \cosh t$  on the interval  $[0, 1]$ .
15. Set up the integrals to find the center of mass for the lamina of density 1 given by the region bounded by  $y = x^2 - x$  and  $y = 2$ .
16. Set up the integral to find the surface area of the barrel given by revolving the curve  $y = \sin x$  on the interval  $[\frac{\pi}{4}, \frac{3\pi}{4}]$  about the  $x$ -axis.
17. Find the work done in stretching a spring an additional foot if a force of 10 pounds is used to stretch the spring one foot.
18. Set up the integral to find the work done in filling an empty cylindrical water tank with radius 5 feet which is 10 feet above the ground from a source at ground level with water of weight  $62.4 \text{ lb/ft}^3$ .
19. If the half life of a radioactive element is 1500 years, set up an equation to find the amount of this radioactive element in a storage site which contains 100 kg of the element now for any time  $t$ .
20. Evaluate the following integrals:

(a)  $\int x^2 e^{x^3} dx$

(b)  $\int \frac{\ln x}{x^2} dx$

(c)  $\int \sqrt{1 - \cos t} dt$

(d)  $\int \frac{4 + 2x}{9 + x^2} dx$

(e)  $\int \sec^3 2x \tan 2x dx$

(f)  $\int \frac{2x + 1}{x(x^2 + 1)} dx$

(g)  $\int \frac{2x + 1}{x^2 - 6x + 8} dx$

(h)  $\int \sin^4 y dy$ .

(i)  $\int \frac{dt}{(9 + t^2)^{\frac{3}{2}}}$

(j)  $\int \frac{t^3 dt}{\sqrt{1 - t^2}}$