Practice Final
MATH 9C

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1. Determine if the following improper integral converges or diverges:

$$\int_{-1}^{1} \frac{2x^2 - x + 2}{x^2(x-2)}\,dx$$

2. Consider the sequence $a_n = \dfrac{3^n + \cos(n\pi)}{2^n}$.

   (a) Is a_n bounded above? If so, find an upper bound.

   (b) Is a_n non decreasing? Explain your answer.

   (c) Find $\lim_{n\to\infty} a_n$.

3. List the first three partial sums of $\sum_{n=1}^{\infty} \dfrac{(-1)^{(n+1)}}{n2^n}$.

4. Suppose a_n and b_n are positive sequences and $\sum_{n=1}^{\infty} a_n + b_n$ converges. Is it true that $\sum_{n=1}^{\infty} a_n$ converges? Why or why not?

5. Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{3(-2)^n}{3^n}$

6. Determine if the following series converge or diverge.

   (a) $\sum_{n=1}^{\infty} \dfrac{n^n}{(n!)^2}$

   (b) $\sum_{n=2}^{\infty} \dfrac{(-1)^n \ln n}{n}$

   (c) $\sum_{n=1}^{\infty} \dfrac{1}{n(n+2)}$

   (d) $\sum_{n=1}^{\infty} \dfrac{\sin n}{3^n}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

(f) $\sum_{n=1}^{\infty} \ln n - \ln(n+1)$

7. Determine if the following series are conditionally convergent, absolutely convergent or divergent:

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{8n+1}}$

   (b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n3^n}$

   (c) $\sum_{n=1}^{\infty} \frac{(4n-1)^n}{(2n+1)^n}$

8. Determine the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}(2x-1)^n$

9. Determine the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{n!}{2^n}(x+4)^n$

10. If $f(x) = \sum_{n=0}^{\infty} a_n(x-2)^n$ has a radius of convergence equal to 0, does $f(2)$ necessarily converge?

11. Find the 2rd Taylor polynomial for $f(x) = \sin(\pi x)$ centered at $x = \frac{1}{4}$.

12. Find the Taylor series for $f(x) = xe^x + 1$ centered at $x = 0$.

13. Find the Maclaurin series for $f(x) = \frac{\cos(x) - 1}{x^2}$.

14. Find the Maclaurin series for $f(x) = \arctan x^3$.

15. Show that $y = e^x - e^{-x}$ is a solution to the differential equation $y' + y = 2e^x$.

16. Sketch the slope fields for $y' = \frac{y}{x}$

17. Find the general solution to $y' = (y^2 - 1)e^x$.

18. Find the solution to the initial value problem $y' + y = e^{-x}\cos x$, $y(0) = 1$.

19. Find the general solution to $x^2y' - y = x$.

20. A 20 gallon container contains 20 gallons of water and 2 pounds of salt. A saline solution with .2 pounds of salt per gallon is poured into the container at a rate of 4 gallons a minute. If the subsequent mixture leaves the container at a rate of 4 gallons a minute, set up a differential equation with initial conditions to solve for the amount of salt in the container at any time t.

21. \$10,000 is invested in a bank account which accrues interest at an annual rate of 4 percent compounded continuously. Find an equation to solve for the amount of money in the account at any time t.

22. A cart is coasting down the road at a rate of 50 meters per minute. If the cart weighs 200 kg and the resistance is given by four times the speed it is traveling, find its velocity after 5 minutes.

23. Find the orthogonal trajectories to $y = cx^2$.

24. Find the series solution to $y'' + y = 0$ with initial conditions $y'(0) = 1$ and $y(0) = 0$.

25. Find the slope of the tangent line to the curve $r = 4\cos(2\theta)$ at the point $(2, \frac{\pi}{6})$.

26. Find the points (in polar form) where the curve $r = 1 - 2\cos\theta$ has a horizontal tangent line.

27. Find the area enclosed by one of the petals of the rose curve $r = 2\cos(3\theta)$.

28. Find the area outside the inner loop and inside the outer loop of the limaçon $r = 1 + 2\sin\theta$.

29. Find the length of one of the petals on the curve $r = 2\cos(2\theta)$.