Practice Final
MATH 9C

Prof. Janet Vassilev

March 12, 2008

1. Determine if the following improper integrals converge or diverge:

\[ \int_{-1}^{1} \frac{1}{x^2} \, dx \]

\[ \int_{1}^{\infty} \frac{1}{\sqrt{x} + 1} \, dx \]

\[ \int_{2}^{\infty} \frac{1}{x(\ln x)^3} \, dx \]

2. Determine the convergence of the following sequences:

(a) \( a_n = \left(1 + \frac{2}{n}\right)^n \)

(b) \( a_n = (-1)^n + 2 \)

(c) \( a_n = \frac{\sin n}{n} \)

3. List the first three partial sums of \( \sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n2^n} \).

4. Suppose \( a_n \) and \( b_n \) are positive sequences and \( \sum_{n=1}^{\infty} a_n + b_n \) converges. Is it true that \( \sum_{n=1}^{\infty} a_n \) converges? Why or why not?

5. Find the sum of the series \( \sum_{n=0}^{\infty} \frac{3(-2)^n}{3^n} \).

6. Determine if the following series converge or diverge.

(a) \( \sum_{n=1}^{\infty} \frac{n^n}{(n!)^2} \)

(b) \( \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n} \)
(c) \( \sum_{n=1}^{\infty} \frac{1}{n(n+2)} \)

(d) \( \sum_{n=1}^{\infty} \frac{\sin n}{3^n} \)

(e) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \)

(f) \( \sum_{n=1}^{\infty} \ln n - \ln(n+1) \)

7. Determine if the following series are conditionally convergent, absolutely convergent or divergent:

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{8n+1}} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-2)^n}{n3^n} \)

(c) \( \sum_{n=1}^{\infty} \frac{(4n-1)^n}{(2n+1)^n} \)

8. Determine the interval of convergence for the power series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}(2x - 1)^n \)

9. Determine the interval of convergence for the power series \( \sum_{n=0}^{\infty} \frac{n!}{2^n}(x + 4)^n \)

10. If \( f(x) = \sum_{n=0}^{\infty} a_n(x - 2)^n \) has a radius of convergence equal to 0, does \( f(2) \) necessarily converge?

11. Find the 2nd Taylor polynomial for \( f(x) = \sin(\pi x) \) centered at \( x = \frac{1}{4} \).

12. Find the Taylor series for \( f(x) = xe^x + 1 \) centered at \( x = 0 \).

13. Find the Maclaurin series for \( f(x) = \frac{\cos(x) - 1}{x^2} \).

14. Find the Maclaurin series for \( f(x) = \arctan x^3 \).

15. Show that \( y = e^x - e^{-x} \) is a solution to the differential equation \( y' + y = 2e^x \).

16. Find the general solution to \( y' = (y^2 - 1)e^x \).

17. Find the solution to the initial value problem \( y' + y = e^{-x} \cos x, \ y(0) = 1 \).

18. Find the general solution to \( x^2y' - y = x \).

19. A 20 gallon container contains 20 gallons of water and 2 pounds of salt. A saline solution with .2 pounds of salt per gallon is poured into the container at a rate of 4 gallons a minute. If the subsequent mixture leaves the container at a rate of 4 gallons
a minute, set up a differential equation with initial conditions to solve for the amount of salt in the container at any time \( t \).

20. $10,000 is invested in a bank account which accrues interest at an annual rate of 4 percent compounded continuously. Find an equation to solve for the amount of money in the account at any time \( t \).

21. A cart is coasting down the road at a rate of 50 meters per minute. If the cart weighs 200 kg and the resistance is given by four times the speed it is traveling, find its velocity after 5 minutes.

22. Find the orthogonal trajectories to \( y = cx^2 \).

23. Find the third Taylor polynomial to approximate the series solution to \( y'' + y = 0 \) with initial conditions \( y'(0) = 1 \) and \( y(0) = 0 \).

24. Find the third Taylor polynomial to approximate the series solution to \( y'' - y' = 1 + x^2 \) with initial conditions \( y'(0) = 2 \) and \( y(0) = -1 \).

25. Find the slope of the tangent line to the curve \( r = 4 \cos(2\theta) \) at the point \( (2, \frac{\pi}{6}) \).

26. Find the points (in polar form) where the curve \( r = 1 - 2 \cos \theta \) has a horizontal tangent line.

27. Find the area enclosed by one of the petals of the rose curve \( r = 2 \cos(3\theta) \).

28. Find the area outside the inner loop and inside the outer loop of the limaçon \( r = 1 + 2 \sin \theta \).

29. Find the length of one of the petals on the curve \( r = 2 \cos(2\theta) \).