

Practice Midterm

MATH 46

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1. What is the order of the following differential equation: $y^{(4)} - 2y' + xy = y^6$?
2. Give a rough sketch of the solution of $y' = y^2 - x$ with initial condition $y(2) = 1$? (Hint: Draw a direction field and sketch the integral curve containing the above point.)
3. Solve the initial value problem $xy' - y = x \ln x$, $y(1) = 4$.
4. Solve $y' = \frac{y^2}{x^2 + 1}$.
5. Solve $y' + y = xe^{2x}y^3$.
6. Solve $y' = \frac{x^2 + y^2}{xy}$.
7. Determine if $(4xy + y^2)dx + (2xy + 2x^2 + y^3)dy = 0$ is exact.
8. Find an integrating factor for $(x^2 + y^2)dx + (2xy + 3xy^2 + x^3)dy = 0$.
9. If the amount of a substance in the air changes at a rate which is inversely proportional to the amount present, write a differential equation to find $Q(t)$, the amount of the substance in the air.
10. A radioactive substance decays at a rate proportional to the amount present. If half the original quantity Q_0 is left after 1000 years, in how many years will the substance decay to one fourth of the original quantity?
11. A thermometer reading 300 degrees is placed in a room where the temperature is 75 degrees. After two minutes the thermometer reads 200 degrees. What does the thermometer read after 10 minutes?
12. An object with mass 98 kg is launched vertically upward with initial velocity of 49 m/s in a medium which exerts a resistive force with magnitude proportional to the square of the speed. Write a differential equation to find the velocity of the object.
13. Find the orthogonal trajectories of $x^2 + 4y^2 = c^2$.