# Analysis and applications of evolutionary PDEs

Inaugural IAMIS workshop at UC Riverside

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ABSTRACTS

## Andrea Bertozzi, UCLA

*Particle laden flow - applications from the Gulf oil spill to spiral concentrators*

This talk is about the challenges in modeling particle laden flow in viscous liquids and some of their applications. Particle settling in liquid is a complex process in which long range interactions are important due to the incompressibility of the fluid. Hinderance in particle settling is well-known as are other multi-particle effects such as shear-induced migration. We look at the basic physics of particle laden flow on an incline and show that this problem has a natural bifurcation between settling-dominated dynamics and dynamics dominated by the shear flow. In both cases the presence of particles greatly changes the flow structure from that of a single clear fluid. We also present some recent work connecting this physics to the modeling of particle separation in a spiral separator—used in the mining industry for separating different densities and sizes of particles in a slurry. Mathematical models include both systems of conservation laws (involving both classical and singular shock solutions) and flows with surface tension.

## Andrew Bernoff, Harvey Mudd

*A Primer of Swarm Equilibria*

We study equilibrium configurations of swarming biological organisms subject to exogenous and pairwise endogenous forces. Equilibrium solutions are extrema of an energy functional, and satisfy a Fredholm integral equation. In one spatial dimension, for a variety of exogenous forces, endogenous forces, and domain configurations, we find exact analytical expressions for the equilibria. The equilibria typically are compactly supported and may contain $\delta$-concentrations or jump discontinuities at the edge of the support. In two dimensions we show that the Morse Potential and other "pointy" potentials lead to inverse square-root singularities in the density at the edge of the swarm support. This is joint work with Louis Ryan of Harvey Mudd and Chad M. Topaz of Macalester College.

**Björn Birnir, UCSB**

*The Kolmogorov-Obukov theory of turbulence*

The Kolmogorov-Obukov statistical theory of turbulence, with intermittency corrections, is derived from a stochastic Navier-Stokes equation with generic noise. In this talk we discuss how the laminar solution of the Navier-Stokes equation becomes unstable for large Reynolds number and the unstable solution is the solution of the stochastic Navier-Stokes equation. This is the unique solution that describes fully-developed turbulence. In order to compare with experiments and simulations, we find the solution of the stochastic Hopf's equation for the invariant measure. Gaussian noise and dissipation intermittency produce the Kolmogorov-Obukov scaling of the structure functions of turbulence. The Feynmann-Kac formula produces log-Poisson processes from the stochastic Navier-Stokes equation. These processes, first found by She, Leveque, Waymire, and Dubrulle in 1995, give the intermittency corrections to the structure functions of turbulence, stemming from velocity fluctuations. The probability density function (PDF) of the two-point statistics that can be compared to experiments and simulations turn out to be similar to the generalized hyperbolic distributions first suggested by Barndorff-Nilsen in 1977. We compare the theoretical PDF with PDFs obtained from DNS simulations and wind-tunnel experiments.

**Gautam Iyer, Carnegie Mellon**

*From homogenization to averaging in cellular flows*

We consider an elliptic eigenvalue problem in the presence a fast cellular flow in a two-dimensional domain. It is well known that when the amplitude, $A$, is fixed, and the number of cells, $L^2$, increases to infinity, the problem "homogenizes"; that is, can be approximated by the solution of an effective (homogeneous) problem. On the other hand, if the number of cells, $L^2$, is fixed and the amplitude $A$ increases to infinity, the solution "averages". In this case, the solution equilibrates along stream lines, and it's behaviour across stream lines is given by an averaged diffusion equation.

In this talk we study what happens if we simultaneously send both the amplitude $A$, and the number of cells $L^2$ to infinity. It turns out that if $A \ll L^4$, the problem homogenizes, and if $A \gg L^4$, the problem averages. The transition at $A \approx L^4$ can quickly predicted by matching the effective diffusivity of the homogenized problem, to that of the averaged problem. However a rigorous proof is much harder, in part because the effective diffusion matrix is unbounded. I will provide the essential ingredients for the proofs in both the averaging and homogenization regimes. This is joint work with T. Komorowski, A. Novikov and L. Ryzhik (with numerics by K. Zygalakis).

**Eun Heui Kim, Cal State Long Beach**

*Transonic problems for multidimensional conservation laws*

We discuss the recent development of mathematical theories on transonic two-dimensional Riemann problems in conservation laws. We discuss a simplified model, the nonlinear wave system, from the compressible gas dynamics. We present both numerical results and analytical results on the model system in various configurations.

**Inwon Kim, UCLA**

*Droplet spreading on a surface with random obstacles*

We consider the spreading of a droplet on a planar surface covered with random obstacles. Assuming the distributions of the obstacles are stationary ergodic and taller than the droplet, we study the homogenized limit. In particular we show the uniform convergence of the droplet interface evolving with the "accelarated" motion law. This is joint work with Nestor Guillen.

**Hailiang Liu, Iowa State**

*On the FENE dumbbell model of polymeric flows*

The FENE dumbbell model consists of the incompressible Navier-Stokes equation and the Fokker-Planck equation for the polymer distribution. In such a model, the polymer elongation cannot exceed a limit, yielding all interesting features near the boundary. In this talk I'll recount progress toward understanding the sharpness of boundary requirement dictated by the elongation parameter, with resolution based on a well-posedness theorem for the coupled Fokker-Planck-Navier-Stokes system. I'll also present an entropy satisfying conservative method to compute the underlying Fokker Planck dynamics.

**Sébastien Motsch, CSCAMM**

*Mathematical modeling of self-organized dynamics: from microscopic to macroscopic description*

In many biological systems, we observe the emergence of self-organized dynamics (e.g. flock of birds, school of fish, aggregation of bacteria). To model these phenomena, we can either use "microscopic models" describing the motion of each individual or "macroscopic models" describing the evolution of the density of individuals. In this talk, we discuss how we can "link" the two approaches using kinetic theory.

In contrast with particle systems in physics, models of self-organized dynamics do not conserve momentum and energy. This lack of conservation requires to introduce new tools to derive macroscopic models. For instance, a new type of "collisional invariant" allows us to derive the hydrodynamic limit of a large class of "microscopic models." The macroscopic models obtained are non-conservative hyperbolic systems. We present a numerical scheme to investigate the behavior of such systems.

**Toan Nguyen, Brown**

*On the stability of boundary layers in the Navier-Stokes equations*

In fluid dynamics, a classical problem is to understand the dynamics of viscous fluid flows past solid bodies, especially in the regime of very high Reynolds numbers (or very small viscosity). Boundary layers are typically formed in a thin layer near the boundary. In this talk, I shall review recent developments on stability of boundary layers (for compressible and incompressible fluids). In particular, I will present some main ingredients of a stability analysis of suction/blowing boundary layers. I will also discuss various ill-posedness results obtained for the classical Prandtl boundary layers and the relevance of boundary-layer expansions in the inviscid limit of the Navier-Stokes equations.

**Vladislav Panferov, Cal State Northridge**

*Phase transitions in models of Vlasov-McKean type*

I will discuss the problem of non-uniqueness and stability of steady states for equations of Vlasov-McKean type. These equations provide a mean-field description of a system of interacting diffusions; particularly we consider the problem with the spatial variable in a periodic box of size $L$ and particles interacting through a pairwise potential $V$. If the Fourier transform of $V$ has a negative minimum, the system has a critical threshold for the diffusion constant beyond which the trivial uniform steady state becomes unstable and the system experiences a phase transition. We show that for a large class of interactions, when the size of the domain is sufficiently large, the transition is always discontinuous and is characterized by coexistence of several stable states in a certain interval of parameter space.The transition is also shown to occur at a value of the diffusion constant strictly greater than the critical threshold. I will then briefly present the results of a numerical study on the character of phase transition in Vicsek like models of flocking, in which a similar discontinuous transition is observed.

**Steve Shkoller, UC Davis**

*On the finite-time splash and splat singularities for the 3-D free-surface Euler equations*

We prove that the 3-D free-surface Euler equations with regular initial geometries and velocity fields have solutions which can form a finite-time “splash” (or “splat”) singularity, wherein the evolving 2-D hypersurface, the moving boundary of the fluid domain, self-intersects at a point (or on a surface). Such singularities can occur when the crest of a breaking wave falls unto its trough, or in the study of drop impact upon liquid surfaces. Our approach is founded upon the Lagrangian description of the free-boundary problem, combined with a novel approximation scheme of a finite collection of local coordinate charts; as such we are able to analyze a rather general set of geometries for the evolving 2-D free-surface of the fluid. We do not assume the fluid is irrotational, and as such, our method can be used for a number of other fluid interface problems, including compressible flows, plasmas, as well as the inclusion of surface tension effects. This is joint work with D. Coutand.

**Thomas Sideris, UCSB**

*Almost global existence of small solutions for 2D incompressible Hookean elastodynamics*

We study the initial value problem for elasticity on all of $R^2$ in the incompressible Hookean case. If the initial displacement and velocity are of order $\varepsilon$ in an appropriate energy norm, for sufficiently small $\varepsilon$, then there exists a smooth solution for a time interval of length greater than $\exp(C/\varepsilon)$, where $C$ is independent of the initial data and $\varepsilon$. The proof combines energy estimates with a ghost weight, decay estimates in $L^2$ and $L^\infty$, and the null structure of the nonlinear terms. This is joint work with Zhen Lei and Yi Zhou of Fudan University.