Mathematical Physics & Dynamical Systems		
Fall 2011		
Date	Speaker	Title of the talk
9-29-2011	Stephen Muir	Gibbs measures for classical lattice models Part 1
10-6-2011	Stephen Muir	Gibbs measures for classical lattice models Part 2
10-13-2011	Richard Niemeyer	Fractal-like characteristics of the brain: a review
10-20-2011	Dr. Tanja Eisner UCLA	Arithmetic progressions and ergodic theory
10-27-2011	Dr. Michel Lapidus	Fractal Strings, Complex Dimensions and the Spectral Operator: From the Riemann Hypothesis to Universality and Phase Transitions
11-3-2011	Robert Niemeyer	
11-10-2011		No seminar
11-17-2011	John Quinn	
11-24-2011		No seminar
12-1-2011	Dominick Scaletta	Dirac Cohomology and its Applications in Physics and Mathematics

Arithmetic progressions and ergodic theory
Dr. Tanja Eisner

University of California, Los Angeles

MPDS seminar

October 20

Abstract

We sketch the development from van der Waerden's theorem on arithmetic progressions to the recent Green-Tao theorem and show how methods from ergodic theory have been decisive in this field.

Fractal Strings, Complex Dimensions and the Spectral Operator:

From the Riemann Hypothesis to Universality and Phase Transitions

Michel L. LAPIDUS

MPDS seminar

October 27

Abstract

In [1] (J. London Math. Soc., 1995), a spectral reformulation of the Riemann hypothesis was obtained by the author and Helmut Maier, in terms of inverse spectral problems for fractal strings. In short, one can always hear whether a given fractal string of dimension D (different from1/2) is Minkowski measurable if and only if the Riemann hypothesis is true. Later on, this work was revisited in light of the theory of complex dimensions of fractal strings developed by the author and Machiel van Frankenhuijsen in [2] (*Fractal* Geometry *and Number Theory,* Birkhauser, 2000) and [3] (*Fractal Geometry, Complex Dimensions and Zeta Functions,* Springer, 2006; 2^{nd} rev. and enl. edn. to appear in 2012). Moreover, in [3], the "spectral operator" was introduced semi-heuristically as the operator that sends the geometry of a fractal string onto its spectrum.

In a forthcoming memoir, joint with Hafedh Herichi, we provide a rigorous functional analytic framework for the study of the spectral operator *a.* We show that *a* is an unbounded normal operator acting on a suitable scale of Hilbert spaces (indexed by the Minkowski dimension D in (0,1) of the underlying fractal strings) and precisely determine its spectrum (which turns out to the be equal to the closure of the range of values of the Riemann zeta function along the vertical line Re s = D). Furthermore, we introduce a suitable family of truncated spectral operators and deduce that for a given D > 0, the spectral operator is quasi-invertible (i.e., each of the truncated spectral operators is invertible) if and only if there are no Riemann zeros on the vertical line Re s = D. It follows that the associated inverse spectral problem has a positive answer for all possible dimensions D in (0,1), other than in the mid-fractal case when D = ½, if and only if the Riemann hypothesis is true. Using results concerning the universality of the Riemann zeta function among the class of non-vanishing holomorphic functions, we also show that the spectral operator is invertible for D > 1, not invertible for ½ < D < 1, and conditionally (i.e., under the Riemann hypothesis), invertible for 0 < D < 1/2. Moreover, we prove that the spectrum of the spectral operator is bounded for D > 1, unbounded for D =1, the entire complex plane for ½ < D < 1, and unbounded but, conditionally, not the whole complex plane, for 0 < D < 1/2.

We therefore deduce that four different types of (mathematical) phase transitions occur for the spectral operator at the critical values D = ½ and D = 1, concerning the shape of its spectrum, its boundedness (bounded for D > 1, unbounded otherwise), its invertibility (with phase transitions

at $D = 1$ and, conditionally, at $D = 1/2$), as well as its quasi-invertibility (with a single phase transition at $D = 1/2$ if and only the Riemann hypothesis holds true) .

We also show that (as was intuitively suggested in [3]) the spectral operator has an operator-valued Euler product that is convergent (in a suitable sense) even in the critical strip $0 < \mathrm{Re}\, s < 1$.

From a philosophical point of view, these new developments allow us to provide a natural quantization of many aspects of analytic number theory; that is, to associate natural operators to L-functions (such as the Riemann zeta function) and to correspondingly quantize (and extend) the identities they satisfy (such as their functional equation and their Euler product representation).

If time permits, and as a further illustration of this principle, we will also discuss a very recent work by the author in which we provide a natural operator-theoretic generalization of Voronin's original Universality Theorem and its various extensions. We may also mention recent progress (made in collaboration with Machiel van Frankenhuijsen and using in an essential manner some of the above results) in the search for a cohomology theory associated with the complex dimensions of a fractal object or of an arithmetic geometry.