Quivers, with a view toward Gabriel's Theorem

(in the Lie Theory Seminar)

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Outline. Quivers are graphs with oriented edges. Their study has applications to finite-dimensional algebras, Kac-Moody Lie algebras, quantum groups, and representation theory.

This mini-course is divided into two parts, with one lecture "in between". The first comprises purely algebraic basics (*2 lectures*) - we will define quivers and their representations, and see several examples. We next introduce the quiver algebra - which has the same representations as above - and study projectives and the "standard resolution" for it.

There will be *one lecture* on combinatorics, where I will sketch a proof of the classification of quivers with positive (semi)definite Euler form. This solves the following problem: suppose some countries have oil pipelines between them, such that the total oil content (in barrels) for any country equals exactly half of the total amount among all its neighbors. What are all possible configurations?

Finally, we get down to the main goal of this course - to state and prove a relation between representation theory (*When is a simply laced quiver algebra of finite type?*) and Dynkin diagrams (*Only when the quiver is of type ADE.*), using geometry (*3 lectures*). This will involve algebraic groups acting on affine varieties (mostly, vector spaces). The first of these lectures addresses the "easy" part of the proof - at most five lines, and where one has more "group actions" than algebraic geometry. The other part is more technical.

References.

(1) Michel Brion, *Lecture Notes on "Representation of Quivers"* (Summer School on Geometric Methods in Representation Theory, Grenoble 2008), available at `http://www-fourier.ujf-grenoble.fr/~mbrion/notes.html`.

(2) Harm Derksen and Jerzy Weyman, *Quiver Representations* (AMS Notices **52** (2005), 200–206), available at `http://www.ams.org/notices/200502/fea-weyman.pdf`.

(3) Victor G. Kac, *Infinite-dimensional Lie algebras*, Chapter 4.

(4) David Eisenbud, *Commutative algebra, with a view towards algebraic geometry* (for the geometry part).