Math 131 - HW 2

Finish reading Chapter 1 (if you haven't already) and read Sections 2.A and 2.B, which we will cover in class next week.

1. Let $V = \{(a_1, a_2) | a_1, a_2 \in \mathbb{F}\}$. If $(a_1, a_2), (b_1, b_2) \in V$ and $s \in \mathbb{F}$, define addition and scalar multiplication as follows:

$$(a_1, a_2) + (b_1, b_2) := (a_1 + b_1, a_2 b_2) \quad \text{and} \quad s(a_1, a_2) := (sa_1, a_2).$$

   Is V a vector space under these operations? Justify your answer.

2. Which of the following are subspaces of $\mathbb{R}^3$? Why or why not?
   (a) $\{(x, y, z) \in \mathbb{R}^3 | x - 4y - z = 0\}$
   (b) $\{(x, y, z) \in \mathbb{R}^3 | 2x^2 - y + z = 0\}$

3. Prove or give a counterexample: The intersection of any two subspaces of V is a subspace.

4. Prove or give a counterexample: The union of any two subspaces of V is a subspace.

5. Prove or give a counterexample: If W_1, W_2 and U are each subspaces of V and $U + W_1 = U + W_2$, then $W_1 = W_2$.

6. Find a subset of $\mathbb{R}^2$ that is closed under scalar multiplication, but is not a subspace.