Math 131 - HW 3

Read Chapter 2.

1. Show that if $v_1, \ldots, v_m$ and $w_1, \ldots, w_n$ are lists of vectors in V, then

$$\text{span}(v_1, \ldots, v_m, w_1, \ldots, w_n) = \text{span}(v_1, \ldots, v_m) + \text{span}(w_1, \ldots, w_m).$$

2. Show that a list u, v of two vectors is linearly dependent if and only if one of the vectors is a multiple of the other.

3. Find three vectors in $\mathbb{R}^3$ which make a linearly dependent list, but none of the three is a multiple of another.

4. Prove or give a counterexample: If $v_1, \ldots, v_m$ and $w_1, \ldots, w_m$ are linearly independent lists of vectors in V, then $v_1 + w_1, \ldots, v_m + w_m$ is linearly independent.

5. Suppose $v_1, \ldots, v_m$ is linearly independent in V and $w \in V$. Show that $v_1, \ldots v_m, w$ is linearly independent if and only if $w \notin \text{span}(v_1, \ldots v_m)$.

6. Suppose v_1, v_2, v_3, v_4 is a basis of V. Prove that $v_1, v_1 + v_2, v_2 + v_3, v_3 + v_4$ is also a basis of V.