Math 131 - HW 4

Read Sections 3.A and 3.B.

1. Suppose that $T \in \mathcal{L}(V, W)$ and $v_1, \ldots, v_n$ is a list of vectors in V.
   (a) Prove or give a counterexample: If $v_1, \ldots, v_n$ is linearly independent, then $T(v_1), \ldots, T(v_n)$ is linearly independent.
   (b) Prove or give a counterexample: If $T(v_1), \ldots, T(v_n)$ is linearly independent, then $v_1, \ldots, v_n$ is linearly independent.

2. Find a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that for any $\lambda \in \mathbb{R}$ and $v \in \mathbb{R}^2$,
   $$f(\lambda v) = \lambda f(v),$$
   but f is NOT a linear map.

3. Find a function $g : \mathbb{C} \to \mathbb{C}$ such that for any $z, w \in \mathbb{C}$ we have
   $$g(z + w) = g(z) + g(w),$$
   but g is NOT a linear map (over $\mathbb{C}$).

4. (a) Let V_1 be a subspace of V. If $T : V \to W$ is a linear map, prove that
   $$T(V_1) = \{T(v) \mid v \in V_1\}$$
   is a subspace of W.
   (b) Let W_1 be a subspace of W. If $T : V \to W$ is a linear map, prove that
   $$\{x \in V \mid T(x) \in W_1\}$$
   is a subspace of V.