Math 131 - HW 6

Read Section 3.C.

1. Let $\beta = (e_1, \ldots, e_n)$ and $\gamma = (e_1, \ldots, e_m)$ be the standard bases for $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively. For the following linear transformations $T : \mathbb{R}^n \to \mathbb{R}^m$, compute $[T]_\beta^\gamma$ the matrix representation of T with respect to the bases β and γ. (Recall that our book uses the notation $\mathcal{M}(T)$ for $[T]_\beta^\gamma$.)
   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (2x + 3y - z, x + z)$.
   (b) $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, a_2, \ldots, a_n) = (a_1, a_1, \ldots, a_1)$.

2. As we did in class, consider the linear map $D : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ given by $D(p) = p'$ (where p' is the derivative of p). Find a basis β of $\mathcal{P}_3(\mathbb{R})$ and a basis γ of $\mathcal{P}_2(\mathbb{R})$ such that the matrix $[D]_\beta^\gamma$ is:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

3. Suppose that V and W are finite-dimensional vector spaces and that T is a linear map from V to W. Given a basis $\beta = (v_1, \ldots, v_n)$ of V, prove that there exists a basis $\gamma = (w_1, \ldots, w_m)$ of W such that the first column of $[T]_\beta^\gamma$ is of the form:
$$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$