# Math 131 - HW 7

Read Section 3.D and 3.E.
Assume throughout that $V$ and $W$ are both finite-dimensional vectors spaces.

1. Fix a vector $v \in V$. Let
$$E = \{T \in \mathcal{L}(V, W) \ : \ Tv = 0\}.$$
   (a) Prove that $E$ is a subspace of $\mathcal{L}(V, W)$.
   (b) Suppose $v \neq 0$. What is $\dim E$?

2. Suppose $\dim V > 1$. Prove that the set of noninvertible operators on $V$ is not a subspace of $\mathcal{L}(V, V)$.

3. Let $T : V \to W$ be a linear map. Suppose that $v_1, \ldots, v_n$ is a basis for $V$. Prove that $T$ is an isomorphism if and only if $T(v_1), \ldots, T(v_n)$ is a basis for $W$.

4. If $T : V \to W$ is a linear map and $U \subset V$ is a subspace, then we write $T|_U : U \to W$ to denote the restriction of $T$ to $U$, meaning the function defined by $T|_U(u) = T(u)$ for any $u \in U$.

   Suppose $T : V \to W$ is a surjective linear map. Prove that there is a subspace $U$ of $V$ such that $T|_U$ is an isomorphism of $U$ onto $W$. (Hint: This is very similar to problem 5 on HW 5.)

5. Suppose $S, T \in \mathcal{L}(V, V)$ are operators on $V$. Prove that $ST = I$ if and only if $TS = I$.