Guidelines for Proofs

A proof is a nothing more than a clear, compelling argument. It may involve symbols, but in many cases a prose paragraph is better. The important thing to keep in mind is that you make a convincing case for what you're asked to show. When you're deciding how much to say or evaluating whether your argument is clear enough, imagine that your audience is a fellow classmate rather than your instructor.

- Begin by understanding exactly what you need to show. For example, if you're supposed to prove the statement, "Every blax is a whatnot," then you first need to know what a blax and a whatnot are.

- Begin your proof by stating what you plan to show. As you go along, explain why each step is a consequence of the previous one.

- Use complete sentences whenever possible.

- Be clear about the logical relationships between different statements. For example, the statement "If A, then B" is logically equivalent to, "If not B, then not A". However, both of these are different from the statement, "If B, then A".

- Working out the argument and writing it up should often be two separate steps. After you have a complete argument, think about the clearest way to communicate it.

- The best way to learn to write proofs is practice with feedback, but you can also learn a lot from others' proofs. Study the arguments in your textbook for lots of examples. Ask yourself what each step accomplishes, and then step back and see how the whole argument develops from the individual pieces.

- One way to structure a proof is by *contradiction*. Suppose that you're asked to prove a statement of the form, "If A, then B". In a proof by contradiction, you begin by supposing that A is true and B is false. Then, you show that this hypothesis leads to an impossible conclusion. This proves that if A is true, B can't be false.

- A common mistake is to assume the conclusion you want to show and then argue that it leads to a true statement. This doesn't prove anything! You should have a clear idea of what result you want to arrive at, but you shouldn't use it as a starting point.

- If asked to prove relationships among sets, there are some standard ways to proceed. Suppose that A and B are sets.

  - If you want to show $A \subset B$, show that if a is in A, then a is also in B.
  - If you want to show that $A = B$, show $A \subset B$ and $B \subset A$.