Math 131, Fall 18
Discussion Section Worksheet 2

Let V be a vector space.

1. Prove the following useful fact from Tuesday that we skipped over for lack of time:

   For every $v \in V$, $0v = 0$.

2. Let $S = \{(a_1, a_2) | a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in S$ and $c \in \mathbb{R}$, let

$$(a_1, a_2) + (b_1, b_2) := (a_1 + a_2, 0)$$

$$\text{and} \quad c(a_1, a_2) = (ca_1, ca_2).$$

   Determine whether or not these addition and scalar multiplication rules make S a vector space. If not, what axiom(s) fail?

3. Let W_1 and W_2 be subspaces of V. Recall $W_1 + W_2 = \{w_1 + w_2 | w_1 \in W_1 \text{ and } w_2 \in W_2\}$.

   (a) Prove that $W_1 + W_2$ is a subspace of V and that it contains both W_1 and W_2.

   (b) Prove that any subspace of V that contains both W_1 and W_2 must also contain $W_1 + W_2$.[1]

4. True/False: The subset $\{(a + bi, ci) \in \mathbb{C}^2 | a, b, c \in \mathbb{R}\} \subset \mathbb{C}^2$ is a subspace of the complex vector space $\mathbb{C}^2$. Why or why not?

[1] In other words, $W_1 + W_2$ is the smallest subspace of V that contains both W_1 and W_2.