Math 131, Fall 18
Discussion Section Worksheet 3

Let V be a vector space.

1. Let $v_1, \dots v_m$ be a list of vectors in V.
   (a) What is $\mathrm{span}(v_1, \dots v_m)$?

   (b) What does it mean for $v_1, \dots v_m$ to be linearly independent?

2. (a) Show that if we consider $\mathbb{C}$ as a vector space over $\mathbb{R}$ (!), then the list $1 + i, 1 - i$ is linearly independent.

   (b) Show that if we consider $\mathbb{C}$ as a vector space over $\mathbb{C}$, then the list $1 + i, 1 - i$ is linearly dependent.

3. Suppose $v_1, \dots v_m$ is linearly independent in V and $w \in V$. Prove that if $v_1+w, \dots v_m+w$ is linearly dependent, then $w \in \text{span}(v_1, \dots v_m)$.

4. Yesterday in class we showed: *If V is spanned by a finite list, then the length of any linearly independent list of V is less than or equal to the length of any list that spans V.* Without doing any computations, determine whether or not:

   (a) The list $(-2, 0, 3, 4), (1, 2, 5, 4), (-15, 2, 1, 1)$ spans $\mathbb{R}^4$.

   (b) The list $(1, 2, -3), (3, 4, 1), (-3, -3, 3), (2, 4, -1)$ is linearly independent in $\mathbb{R}^3$.

   Hint: Recall that $e_1 = (1, 0, \dots, 0), \dots, e_n = (0, \dots, 0, 1)$ is a linearly independent list that spans V.