Math 131, Fall 18
Discussion Section Worksheet 5

1. For the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$, explain why T is NOT a linear map.
   (a) $T(x, y) = (1, y)$

   (b) $T(x, y) = (x, x^2)$

   (c) $T(x, y) = (\sin x, 0)$

   (d) $T(x, y) = (|x|, y)$

   (e) $T(x, y) = (x + 1, y)$

2. (a) Prove that there exists a linear map $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that

$$T(1,1) = (1,0,2) \text{ and } T(2,3) = (1,-1,4).$$

What is $T(8,11)$?

(b) Is there a linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,6) = (2,1)$?

3. Let V and W be vector spaces. Show that if $T : V \to W$ is linear map, then $T(0) = 0$ (i.e., T send the zero vector in V to the zero vector in W.)

4. (If you have extra time...) Suppose that $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m)$. Show that there is a matrix $A = (A_{i,j})$ where $A_{i,j} \in \mathbb{F}$ for $i = 1,\dots,m$ and $j = 1,\dots,n$ such that for every $(x_1,\dots,x_n) \in \mathbb{F}^n$,

$$T(x_1,\dots,x_n) = (A_{1,1}x_1 + \ldots + A_{1,n}x_n, \dots, A_{m,1}x_1 + \ldots + A_{m,n}x_n).$$

(Hint: You need to find the matrix A. Consider the standard basis $e_1,\dots,e_n$ of $\mathbb{F}^n$. What does T do to this basis?)