Math 131, Fall 18
Discussion Section Worksheet 7

1. Let $\beta = (e_1, \dots, e_n)$ be the standard basis of $\mathbb{R}^n$. For the following linear maps $T : \mathbb{R}^n \to \mathbb{R}^n$, compute the matrix $[T]_\beta^\beta$.
   (a) $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, a_2, \dots) = (a_1, 2a_2, \dots, na_n)$.

   (b) $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, a_2, \dots) = (a_n, a_{n-1}, \dots, a_1)$.

2. Recall the set of $(m \times n)$-matrices $\mathrm{Mat}_{m,n}(\mathbb{F})$ is a vector space under entry-wise addition and scalar multiplication. Consider the basis α of $\mathrm{Mat}_{2,2}(\mathbb{R})$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

   and the basis $\beta = (1, x, x^2)$ of $\mathcal{P}_2(\mathbb{R})$. Define $T : \mathcal{P}_2(\mathbb{R}) \to \mathrm{Mat}_{2,2}(\mathbb{R})$ by

$$T(p) = \begin{bmatrix} p'(0) & 2p(1) \\ 0 & p''(3) \end{bmatrix}$$

   Compute $[T]_\beta^\alpha$.

3. Suppose V is n-dimensional, W is m-dimensional and $T : V \to W$ is a linear map. Let $\beta = (v_1, \dots, v_n)$ be a basis of V. Show that if T is injective, then there is a basis γ of W such that $[T]_\beta^\gamma$ is the $m \times n$-matrix of the form:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & 1 \\ 0 & \dots & 0 & 0 \\ \vdots & & & \vdots \\ 0 & \dots & 0 & 0 \end{bmatrix}.$$

4. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given in the standard basis by the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \in \mathrm{Mat}_{2,3}(\mathbb{R})$$

Find a basis $\beta = (v_1, v_2, v_3)$ of $\mathbb{R}^3$ such that with respect to the basis β of $\mathbb{R}^3$ and the standard basis $\gamma = ((1,0),(0,1))$ of $\mathbb{R}^2$, the matrix of T takes the form

$$[T]_\beta^\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$