1. Let $V$ and $W$ be vectors spaces and $T : V \to W$ be a linear map. Show that if $T$ is a injective and $v_1, \ldots, v_k$ is a linearly independent list of vectors in $V$, then $Tv_1, \ldots, Tv_k$ is a linearly independent list in $W$.

**Proof:** Suppose $a_1, \ldots, a_k \in \mathbb{F}$ and that

$$a_1Tv_1 + \cdots + a_kTv_k = 0.$$ 

To show that $Tv_1, \ldots, Tv_k$ is a linearly independent list, we must show that this implies $a_1 = \cdots = a_k = 0$.

By the linearity of $T$:

$$T(a_1v_1 + \cdots + a_kTv_k) = 0.$$ 

Thus, $a_1v_1 + \cdots + a_kTv_k \in \text{null}(T)$. As $T$ is injective, this implies:

$$a_1v_1 + \cdots + a_kTv_k = 0.$$ 

But $v_1, \ldots, v_k$ is linearly independent, so we can conclude that $a_1 = \cdots = a_k = 0$, as was to be shown.

2. Let $T \in \mathcal{L}(\mathbb{C}^3)$. Suppose that $T$ has eigenvectors $(1, 1, 0)$, $(1, -1, 0)$, $(0, 0, i)$ with respective eigenvalues $2$, $4 + i$ and $-1$.

(a) Is $T$ normal? Why or why not?

Yes! After normalizing the given eigenvectors, we obtain an orthonormal basis of $\mathbb{C}^3$ consisting of eigenvectors for $T$. The complex spectral theorem tells us that this implies that $T$ is normal.

(b) Is $T$ self-adjoint? Why or why not?

No. Self-adjoint operators have only real eigenvalues.

Note: A common mistake was to consider a matrix with columns (or rows) given by the eigenvectors. Note that such a matrix does not represent that operator $T$ (e.g., $(1, 1, 0)$ is not an eigenvector of such a matrix).