Homework Three

Integral Calculus for Life Sciences

UCR Math-007B-B01, Summer 2019

Remember that WolframAlpha can calculate many antiderivatives, allowing you to check your answers to some of these homework exercises.

www.wolframalpha.com

1. (SUBSTITUTION) Evaluate each of the following integrals with the aid of the method of *substitution*. More exercises like this can be found in Neuhauser Chapter 7.1.

$$\int 2x\sqrt{x^2+4}\,\mathrm{d}x \qquad \int 5\cos(1-2x)\,\mathrm{d}x \qquad \int \mathrm{e}^x \sin(\mathrm{e}^x)\,\mathrm{d}x$$

$$\int \mathrm{e}^{2x+3}\,\mathrm{d}x \qquad \int t\mathrm{e}^{-\frac{1}{2}t^2}\,\mathrm{d}t \qquad \int \frac{x+2}{x^2+4x}\,\mathrm{d}x$$

$$\int \frac{3x}{7x+4}\,\mathrm{d}x \qquad \int \sqrt{x+5}\,\mathrm{d}x \qquad \int (3x-2)^{\frac{1}{5}}\,\mathrm{d}x$$

$$\int \frac{x^2-1}{x^3-3x+1}\,\mathrm{d}x \qquad \int \cos(\theta)\mathrm{e}^{\sin(\theta)}\,\mathrm{d}\theta \qquad \int \sec(\mathrm{e}x)\tan(\mathrm{e}x)\,\mathrm{d}x$$

$$\int \tan(x) \qquad \int \sin\left(\frac{3\pi}{2}\theta+\frac{\pi}{4}\right)\mathrm{d}\theta \qquad \int_0^3 t\sqrt{t^2+1}\,\mathrm{d}t$$

$$\int_0^2 \frac{2y}{(2y^2+3)^{1/3}}\,\mathrm{d}y \qquad \int_0^{\pi/4} \tan(x)\sec^2(x)\,\mathrm{d}x \qquad \int_{\ln 4}^{\ln 7} \frac{\mathrm{e}^x}{(\mathrm{e}^x-3)^2}\,\mathrm{d}x$$

2. (INTEGRATION BY PARTS) Evaluate each of the following integrals with the aid of *integration by parts*. More exercises like this can be found on the internet or in Neuhauser Chapter 7.2.

$$\int x\cos(x)\,\mathrm{d}x \qquad \int t^2\mathrm{e}^t\,\mathrm{d}t \qquad \int_0^{\pi/3} \mathrm{e}^{3x}\sin(x)\,\mathrm{d}x$$

$$\int_1^2 x\csc^2(x)\,\mathrm{d}x \qquad \int \ln(t)\,\mathrm{d}t \qquad \int \ln(\sqrt{t})\,\mathrm{d}t$$

3. (POLYNOMIAL LONG DIVISION) Evaluate each of the following integrals by rewriting the integrand using *polynomial long division*. More exercises like this can be found on the internet or in Neuhauser Chapter 7.3.

$$\int \frac{x^3+3x^2+3x-1}{x-3}\,\mathrm{d}x \qquad \int \frac{x^6+7x^3+x+1}{3x+7}\,\mathrm{d}x \qquad \int \frac{x^8-1}{x-1}\,\mathrm{d}x$$

4. *Complete the square* in the denominator of each of these rational functions.

$$\frac{1}{x^2-2x+1} \qquad \frac{1}{x^2+2x+5} \qquad \frac{1}{x^2-16x+64}$$

5. (PARTIAL FRACTION DECOMPOSITION) Evaluate each of the following integrals by first rewriting the integrand using *partial fraction decomposition*. More exercises like this can be found on the internet or in Neuhauser Chapter 7.3.

$$\int \frac{1}{x(x-2)}\,\mathrm{d}x \qquad \int \frac{1}{(x^2-x-2)^2}\,\mathrm{d}x$$

$$\int \frac{4x^2-x-1}{(x+1)^2(x-3)}\,\mathrm{d}x \qquad \int \frac{3x^3+7x^2+5x+3}{(x+1)^2(x^2+1)}\,\mathrm{d}x$$

6. Evaluate these integrals, and notice that although they look very similar, the techniques that can be used to evaluate them are quite different.

$$\int \frac{1}{x^2+1}\,\mathrm{d}x \qquad \int \frac{x}{x^2+1}\,\mathrm{d}x \qquad \int \frac{x^2}{x^2+1}\,\mathrm{d}x \qquad \int \frac{x^3}{x^2+1}\,\mathrm{d}x$$

7. (INTEGRAL DRILLS) In general, when presented with an integral there will be no one to tell you which technique(s) will help you evaluate it, so it's important that you develop the skill of looking at an integral and deciding where to start, and that you become comfortable *playing* with an integral and trying different techniques until something gives. Evaluate each of the following integrals by whatever technique(s) you find that works.

$$\int_1^4 \ln\left(\sqrt{x}+1\right) \qquad \int \frac{3t}{3-t^2}\,\mathrm{d}t \qquad \int \sec(x)\,\mathrm{d}x$$

$$\int \frac{1}{x^2-2x-3}\,\mathrm{d}x \qquad \int \cos(\theta)\sin^3(\theta)\,\mathrm{d}\theta \qquad \int (\ln(x))^3\,\mathrm{d}x$$

$$\int_0^7 \sqrt{49-y^2}\,\mathrm{d}y \qquad \int \frac{1}{x}\csc^2(\ln(x))\,\mathrm{d}x \qquad \int \tan^2(x)\,\mathrm{d}x$$

$$\int \ln(x^2)\,\mathrm{d}x \qquad \int \frac{x^{11}+x+1}{x+11}\,\mathrm{d}x \qquad \int_0^{\pi/6} \mathrm{e}^x\cos(x)\,\mathrm{d}x$$

$$\int \sec^3(t)\,\mathrm{d}t \qquad \int \frac{\sqrt[5]{x^3}+\sqrt[6]{x}}{\sqrt{x}}\,\mathrm{d}x \qquad \int x^3 \mathrm{e}^{x^2}\,\mathrm{d}x$$

$$\int 3y^2\mathrm{e}^{2-y^3}\,\mathrm{d}y \qquad \int \frac{1}{x}\ln(\ln(x))\,\mathrm{d}x \qquad \int \frac{x^3-x^2+x-4}{(x^2+1)(x^2+4)}\,\mathrm{d}x$$

$$\int \frac{1}{\tan(t)}\,\mathrm{d}t \qquad \int_1^2 \mathrm{e}^{\sqrt{x+1}}\,\mathrm{d}x \qquad \int 4t^2\sin(2t^3)\,\mathrm{d}t$$

$$\int x^2\sin(x)\,\mathrm{d}x \qquad \int \frac{1}{x^2-4x-9}\,\mathrm{d}x \qquad \int \frac{x^4+4x^3+6x^2+4x+1}{x^2+x+1}\,\mathrm{d}x$$

8. (Tougher Integrals) Evaluate each of the following integrals.

$$\int \ln\left(x+\sqrt{x}\right)\,\mathrm{d}x \qquad \int \frac{1}{1+\sin(x)}\,\mathrm{d}x \qquad \int_0^{\pi} \sec^2(\theta)\,\mathrm{d}\theta$$

9. (Challenging Integrals) Evaluate each of the following integrals.

$$\int \sqrt{\tan(\theta)}\,\mathrm{d}\theta \qquad \int \sec^3(x)\,\mathrm{d}x \qquad \int \sqrt{\frac{x-1}{x+1}}\cdot\frac{1}{x^2}\,\mathrm{d}x$$

10. (Challenge) Here's an elementary way to estimate the value of π.

   (a) First use polynomial long division to write $\frac{x^4(1-x^4)}{1+x^2}\,\mathrm{d}x$ as a polynomial plus a proper fraction.

   (b) Show that

$$\int_0^1 \frac{x^4(1-x^4)}{2}\,\mathrm{d}x \quad \le \quad \int_0^1 \frac{x^4(1-x^4)}{1+x^2}\,\mathrm{d}x \quad \le \quad \int_0^1 \frac{x^4(1-x^4)}{1}\,\mathrm{d}x.$$

   (c) If you haven't already, evaluate those integrals in the last part, and conclude

$$3.140 \le \pi \le 3.142.$$

11. (MICROTUTORIAL) A population of panda bears in a bio-dome grows at a rate of $P'(t) = 500te^{-\frac{1}{5}t^2}$ where $P(t)$ is the population after t months. If the population right now is 2000, find an equation for $P(t)$ that models what the population will be t months from now. About what will the size of the population be in 3 months? To see an example similar to this one worked through, watch

blendedmathematics.ucr.edu/oil_leak.html

12. (MICROTUTORIAL) The concentration of a protein in a liver cell as a function of time, measured in minutes since the introduction of the protein into the body, is given by the equation

$$p(t) = \frac{1}{2}\left(1 - e^{-t}(\cos(t)\sin(t))\right).$$

The bioavailability of this protein is defined as the integral of this concentration over time. What is the bioavailability of the protein over the first minute of it being introduced? To see an example similar to this one worked through, watch

blendedmathematics.ucr.edu/rumen.html

13. (MICROTUTORIAL) One method of slowing the growth of an insect population without using pesticides is the introduce a number of sterile males that mate with the fertile females but produce no offspring. Let P represent the number of female insects in a population and S the number of sterile males introduced each generation. Let r be the per capita rate of production of females by females, provided their chosen mate is not sterile. Then the population is related to time t, measured in days, by the integral

$$\int \frac{3S\,dP}{P\big((r-1)P - S\big)}.$$

Suppose an insect population with 1000 females grows at a rate of $r = 3$ members per day and 80 sterile males are added. Evaluate the integral to give an equation relating the female population to time. Note that the resulting equation can't be solved explicitly for P. To see an example similar to this one worked through, watch

blendedmathematics.ucr.edu/mosquito_population.html

14. (SAMPLE MIDTERMS) We aren't having a midterm in this course, but note that

on the webpage for the course I've posted links to a few of UCR's usual sample midterms for Math7B, including solutions, just in case you want more practice.

math.ucr.edu/~mpierce/7b

My Solutions to Exercise 6

It's $\arctan(x) + C$.

Use the substitution $u = x^2 + 1$.

Write $\frac{x^2}{x^2+1} = \frac{x^2+1-1}{x^2+1} = 1 + \frac{1}{x^2+1}$.

Do polynomial long division before integrating and get $\frac{1}{2}x^2 + \ln(x-1) + C$.