~~Email Schultz about Quiz.~~

History Notes 6

Move office Hours to 264

- Go over that midterm question ✓
- Make histogram of midterm ✓
- Inform kids that material will get tougher and Schultz says to show up...

1. Suppose $p, q \in \mathbb{Z}_{>0}$ s.t. $p < q$, and suppose $\frac{p}{q}$ has an Egyptian fraction expansion of length k:

$$\frac{p}{q} = \frac{1}{n_1} + \cdots + \frac{1}{n_k}$$

Wlog $n_1 < n_2 < \cdots < n_k$. Show $\frac{p}{q}$ has a length $k+2$ expansion.

[HINT: $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$]

((
- Don't use induction
- Use Schultz's HINTS!
- Don't substitute all the 1's
))

$$\frac{p}{q} = \frac{1}{n_1} + \cdots + \frac{1}{n_k} = \frac{1}{n_1} + \cdots + \frac{1}{n_k}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right) = \cdots$$

2. A wolf, a goat, and a cabbage must be moved across a river by a raft which only has room for one of these and the person steering the raft. If the wolf and goat are left alone the wolf will eat the goat. Similarly if the goat and cabbage are left alone the goat will eat the cabbage. How can someone successfully transport the wolf, goat, and cabbage across the river? goat over, come back cabbage over goat back...

3. Prove this result of Jordanus de Nemore: Given $X, r_1, \ldots, r_{n-1} \in \mathbb{R}^+$ there exists unique $X_1, \ldots, X_n \in \mathbb{R}^+$ s.t. $X = X_1 + \cdots + X_n$ and $X_{i+1}/X_i = r_i \ \forall i \in \{1, \ldots, n-1\}$. Let $S = 1 + r_1 + r_1 r_2 + \cdots + r_1 \cdots r_{n-1}$. Then $X_i = \frac{X}{S} \prod_{j=0}^{i-1} r_j$ ∴

4. Fibonacci: Prove that if the sum of two consecutive integers is a square then the square of the larger integer will be the sum of two (nonzero) squares.

$$-(n + (n-1)) = -u^2 n$$

$$n^2 - (2n-1) = -u^2 + n^2$$

$$(n-1)^2 = -u^2 + n^2$$

$$(n-1)^2 + u^2 = n^2$$ ∎

5. Show that, for F_i being the i^{th} Fibonacci number,

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

(HINT: $F_n^2 = F_n(F_{n+1} - F_{n-1}) = F_n F_{n+1} - F_n F_{n-1}$)