Hist Notes 7

5 1. Recall Nap.log(N) is the number n such that $N = 10^7(1-10^{-7})^n$. Give me a formula for Nap.log(MN) in terms of Nap.log(N) and Nap.log(M).

$MN = 10^7(1-10^{-7})^{m+n}10^7 = 10^7(1-10^{-7})^{m+n}(1-10^{-7})^{-k}$ where $k = $ Naplog 1, $1 = 10^7(1-10^{-7})^k$

$= \cancel{\text{Nap}}\ m+n-k$

$=$ Nap.log M + Naplog N − Naplog 1

4 2. For $c, d \in \mathbb{R}^+$, prove that $x^3+cx=d$ has precisely one real root, and that the root is positive.

$x^3+cx-d = (x-r)(x^2+sx+t)$, and we'll need to show $4t > s^2$ and that $s=r$ is positive. ☺

3 3. Suppose I tell you $\cos(20°)$ is a root of $8y^3-6y-1$. Find a nontrivial cubic polynomial with integer coefficients that has $\cos(40°)$ as a root. (HINT: double-angle formulas)

2 4. Find a nontrivial quartic polynomial ~~that~~ with integer coefficients that has $\sqrt{3-2\sqrt{2}}$ as a root.

1 5. (i) Prove that the sum of the three roots of x^3+px+q, for $p, q \in \mathbb{R}$ is zero.

(ii) Suppose that r and s are positive real roots of $x^3+d=cx$, for $d, c \in \mathbb{R}$. Prove that $r+s$ is a root of $x^3=cx+d$.