NT Discussion Exercises 1.

Tough ones:

1. "Find the units digit of the number 17^{2019}."
   Maybe a warmup to this one...

   "Remainder of 37^{102} when divided by 12" HW

3. "Prove by induction that $\forall n \in \mathbb{N}$ $9 \mid (4^n + 15n - 1)$."

3. (i) Prove that $\forall n \in \mathbb{Z}$, $n^2 \neq 3k+2$ for any $k \in \mathbb{Z}$. HW
   (ii) Prove that if $3 \mid (a^2+b^2)$ for $a, b \in \mathbb{Z}$, then $3 \mid a$ and $3 \mid b$.
   (iii) Prove that in a right triangle with integer sidelengths, ~~one~~ the length of at least one leg must be divisible by three.

4* Let A be an arbitrary 1972 digit number divisible by 9.
   Let a be the sum of the digits of A
   b be " " " a
   c " " " b.
   Prove that c doesn't depend on A. What must c be?

   Euclidean Algorithm?

5. Prove that if a six digit number is of the form $\overline{abcabc}$ with digits a, b, c, then $\overline{abcabc}$ is divisible by 7, 11, 13.