

NT Exercises 3

1. Find all integer solutions to $x^3 + 91 = y^3$.

2. (Homework) Prove that $x^2 - 2y^2 + 12y = 224$ has no integer solutions. (Hint: reduce modulo 8)

3. Solve the system of linear congruences

$$(i) \begin{cases} 2x \equiv 5 \pmod{7} \\ 3x \equiv 4 \pmod{8} \end{cases} \quad (ii) \begin{cases} 4x \equiv 2 \pmod{6} \\ 3x \equiv 5 \pmod{8} \end{cases}$$

Talk about this jump!

4. Characterize all integers x such that x leaves a remainder of 1, 2, 3, 4 when divided by ~~2, 3, 5, 7~~ ~~3, 5, 7, 11~~, respectively.

Smallest positive integer

Just explained this the whole time.

Chinese Remainder Theorem: Given simultaneous congruences $\{x \equiv_{m_i} a_i\}_{i \in I}$, then there exists a unique solution $x \pmod{\prod m_i}$ if the m_i are coprime (and the solution has a form!)

squares (4, 8)

cubes (1, 9, 13)

x^4 (4, 8, 16)

x^5 (1)

Link this to Lagrange Interpolation!

Chinese Remainder Thm
reduction mod m

* \rightarrow
 * Prove
 no solution...