# Final Exam

Ordinary Differential Equations

UCR Math-046-E01, Summer 2018

NAME: _____

STUDENT ID: _____

Please silence your phone during the exam.

Have your ID ready when you turn in your exam to me.

You may use the blank side of any page as scratch paper, but note that I will only be looking for you response to a question on the one side of the page that contains that question.

This exam is long, but that's okay. I don't expect you to respond to *all* the questions in the allotted time. The exam is long so that you have options as to which questions you answer and which you don't answer.

Each page of this exam will be weighted equally when being graded.

Remember that the purpose of this exam is to provide me a document to read so that I may assess how much you've learned in the course, and assess how prepared you are the classes for which this class is a prerequisite. I'm not a grading robot, looking only for right answers. I'm just trying to figure out what you know. So please communicate to me with your responses. Even if you aren't entirely sure about a question, let me know what you *do know* about it. Anything to give me evidence that you've learned something.

Final

1. What is the definition of a *differential equation*?

2. Imagine that your friend is taking this class next quarter and they are completely confused as to how to solve a *Bernoulli* differential equation, but they're totally fine solving *first-order linear* differential equations. Help your friend. Write down the general form of a *Bernoulli* differential equation, and explain to your friend the process of solving one.

Final

3. Give me an example of a single-variable function such that the product of that function with its derivative is equal to two.

4. Give me an example of a single-variable function such that the sum of the function, its first derivative, and its second derivative is equal to zero.

Final

5. Solve one of the following differential equations by whatever method you find that works. Please circle the one you choose to solve.

$y + 2y\cos(2xy) + (x + 2x\cos(xy))\dot{y} = 0$ $\qquad\qquad$ $\ddot{y} - 4\dot{y} + 3y = 2e^{x}$

Final

6. Solve one of the following initial value problems by whatever method you find that works. Please circle the one you choose to solve.

$$x\,y' = y\ln\left(\tfrac{y}{x}\right) \quad \text{where } y(\mathrm{e}) = \mathrm{e}^{\mathrm{e}+2} \text{ and } x > 0$$

$$x^x\,\dot{y} + x^x \ln(x)\,y = 1 \quad \text{where } y(1) = \mathrm{e}^2 - 1 \text{ and } x > 0$$

Final

7. Suppose that $e^x$ and $x$ are complementary solutions to the differential equation

$$y'' + \frac{x}{1-x}y' - \frac{1}{1-x}y = \sec(x)\,.$$

Using the method of *variation of parameters,* calculate the general solution to this differential equation. You may write the particular solution in terms of one or more indefinite integrals.

Final

8. Solve one of the following differential equations by whatever method you find that works. Please circle the one you choose to solve.

$$xy' - 27y' = 3(y-9) \qquad\qquad (ty)^2 + 2y^2 + \dot{y} = 0$$

Final

9. Solve one of the following differential equations by whatever method you find that works. Please circle the one you choose to solve.

$$\frac{\ddot{y}}{y} + \frac{\dot{y}}{y} = 2 \qquad\qquad \dddot{y} + \ln(y) = y\dot{y}$$

Final

10. Find the general solution to the following differential equation using the method of *undetermined coefficients*.

$$y' - 5y = 3e^{2x} - 2x + 1$$

Final

11. For a function of $f(t)$, what is the definition of the Laplace transform $\mathscr{L}\{f(t)\}$?

12. Given the initial value problem of solving $y' + 2y = t\sin(4t)$ where $y(0) = 8$, find an expression for $\mathscr{L}\{y\}$. I'll write some (hopefully) helpful Laplace transforms on the board.

Final

13. Compute the Laplace transform of the following function. I will write some (hopefully) helpful Laplace transforms on the board.

$$g(t) = \begin{cases} t^2 & 0 \le t < 2 \\ e^{3t-6} + 4t - 4 & 2 < t \end{cases}$$

Final

14. A tank initially holds 10 gallons of salt-water solution containing 7 lb of dissolved salt. At $t = 0$ another salt-water solution containing 1 lb of salt *per gallon* is poured into the tank at a rate of 3 gal/min, while the well-stirred mixture leaves the tank at the same rate. Write down a function $Q(t)$ that returns the amount of salt in the tank at any time $t$.

Final

15. A population of wasps in a hive will grow at a rate that is proportional to its current population. In the absence of any outside factors the wasp population will grow by a factor of 7 every 5 days. On any given day about 22 wasps are killed by humans that don't want to get stung, and 18 wasps get lost looking for food and can't find their way back to the hive. If there are initially 100 wasps in the hive, will the hive survive or eventually die out? NOTE: ln(7) is slightly less than 2.

Final